Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.7-P-x-d+e-x^2-^q-a+b-x^2+c-x^4-^p

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [42]. This is test number [44].

1.1   Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.

2. Rubi 4.16.1 in Mathematica 12 on windows 10.

3. Maple 2020 (64 bit) on windows 10.

4. Maxima 5.43 on Linux. (via sagemath 8.9)

5. Fricas 1.3.6 on Linux (via sagemath 9.0)

6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.

7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2   Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (42)	% 0. (0)
Mathematica	% 90.48 (38)	% 9.52 (4)
Maple	% 95.24 (40)	% 4.76 (2)
Maxima	% 0. (0)	% 100. (42)
Fricas	% 21.43 (9)	% 78.57 (33)
Sympy	% 14.29 (6)	% 85.71 (36)
Giac	% 9.52 (4)	% 90.48 (38)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	14.29	0.	76.19	9.52
Maple	23.81	33.33	38.1	4.76
Maxima	0.	0.	0.	100.
Fricas	14.29	7.14	0.	78.57
Sympy	0.	0.	14.29	85.71
Giac	7.14	2.38	0.	90.48

The following is a Bar chart illustration of the data in the above table.

Antiderivative Grade distribution for each CAS

Numbers shown on bars are total percentage solved for each CAS

Rubi	100%
Mathematica	90.48%
Maple	95.24%
FriCAS	21.43%
Giac/Xcas	9.52%
Maxima	0%
Sympy	14.29%

Legend: A, B, C, F
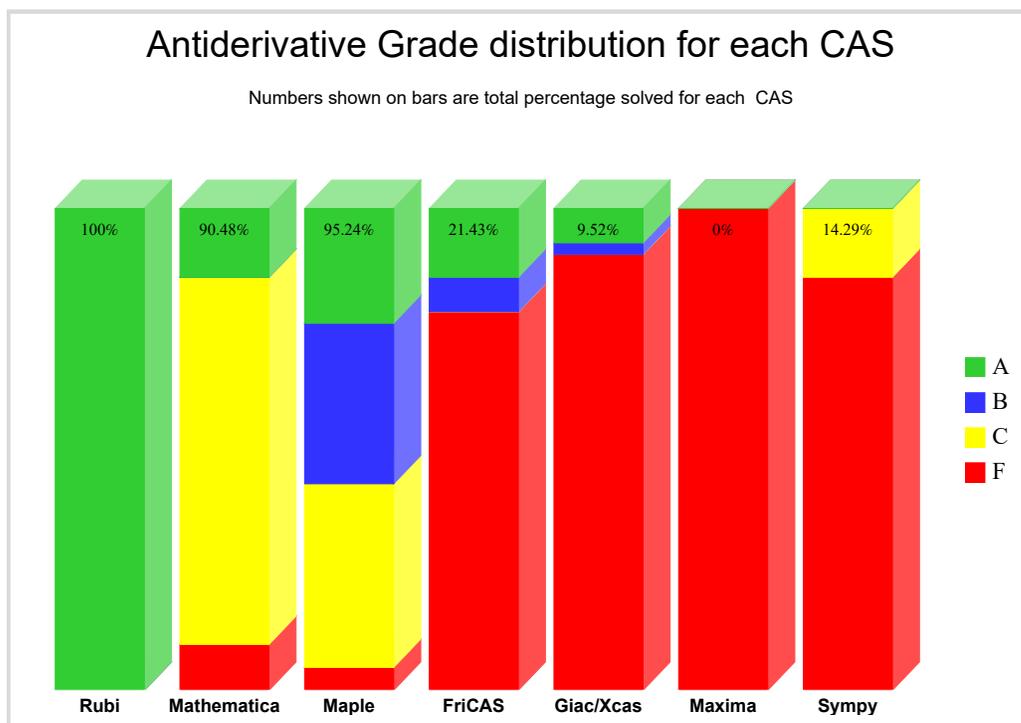
The figure below compares the CAS systems for each grade level.
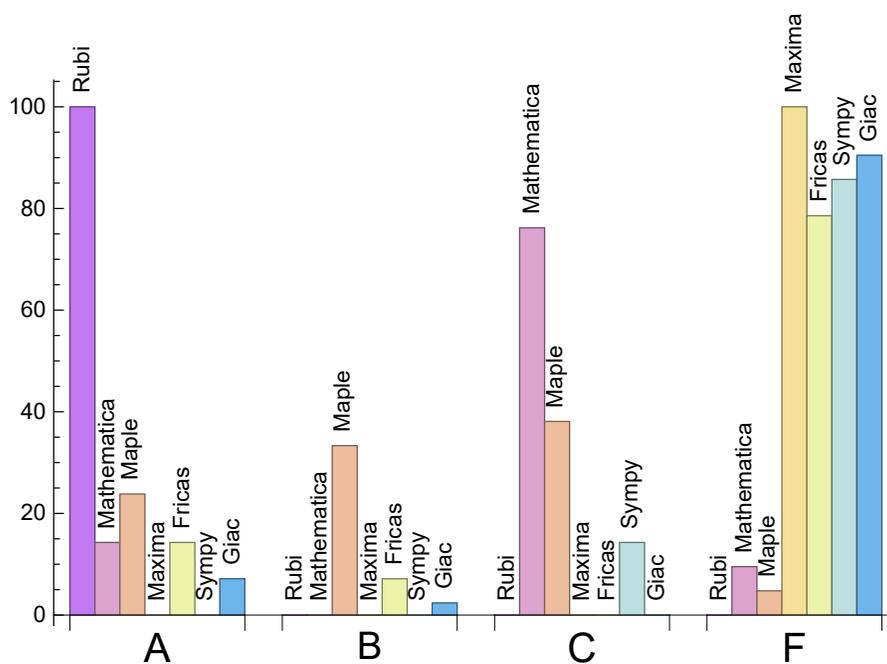
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size
Rubi	0.81	576.24	1.08	396.5
Mathematica	1.68	824.05	1.3	297.5
Maple	0.02	1058.32	1.7	450.
Maxima	Round[Mean[], 0.01]	Round[Mean[], 0.01]	Round[Mean[], 0.01]	Round[Median[], 0.0
Fricas	8.64	1386.22	5.83	1364.
Sympy	6.82	186.	0.53	167.
Giac	1.26	579.	1.97	493.

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {23, 29}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A <u>time limit</u> of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.

POST PROCESSOR SCRIPT
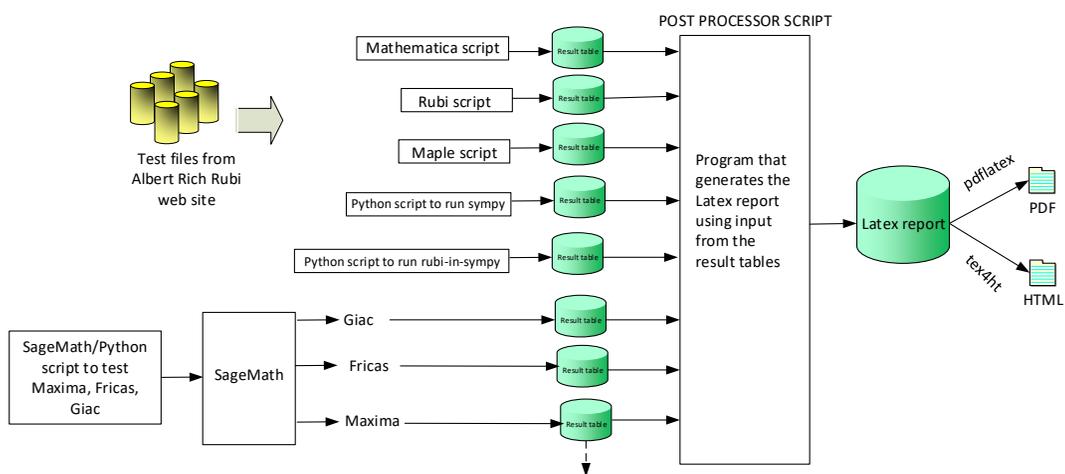
One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 37, 38, 39, 40, 41, 42 }

B grade: { }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31, 32, 34, 35, 36 }

F grade: { 15, 26, 27, 33}

2.1.3 Maple

A grade: { 20, 21, 30, 31, 37, 38, 39, 40, 41, 42 }

B grade: { 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 34, 35, 36 }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 32 }

F grade: { 15, 33 }

2.1.4 Maxima

A grade: {

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

2.1.5 FriCAS

A grade: { 37, 38, 39, 40, 41, 42 }

B grade: { 34, 35, 36 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33 }

2.1.6 Sympy

A grade: { }

B grade: { }

C grade: { 1, 2, 3, 4, 10, 11 }

F grade: { 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

2.1.7 Giac

A grade: { 37, 38, 41 }

B grade: { 42 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	878	217	533	0	0	364	0
normalized size	1	1.94	0.48	1.18	0.	0.	0.8	0.
time (sec)	N/A	0.509	0.282	0.054	0.	0.	6.844	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	706	159	403	0	0	262	0
normalized size	1	1.92	0.43	1.1	0.	0.	0.71	0.
time (sec)	N/A	0.396	0.171	0.008	0.	0.	4.888	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	432	121	269	0	0	167	0
normalized size	1	1.56	0.44	0.97	0.	0.	0.6	0.
time (sec)	N/A	0.225	0.099	0.004	0.	0.	3.291	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	77	169	0	0	78	0
normalized size	1	1.	0.34	0.75	0.	0.	0.35	0.
time (sec)	N/A	0.066	0.034	0.005	0.	0.	1.591	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	138	192	0	0	0	0
normalized size	1	1.	0.37	0.52	0.	0.	0.	0.
time (sec)	N/A	0.36	0.304	0.033	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	641	641	297	679	0	0	0	0
normalized size	1	1.	0.46	1.06	0.	0.	0.	0.
time (sec)	N/A	1.189	1.238	0.025	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	875	875	453	1591	0	0	0	0
normalized size	1	1.	0.52	1.82	0.	0.	0.	0.
time (sec)	N/A	1.784	2.292	0.027	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	912	912	222	588	0	0	0	0
normalized size	1	1.	0.24	0.64	0.	0.	0.	0.
time (sec)	N/A	0.726	0.286	0.036	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	166	458	0	0	0	0
normalized size	1	1.	0.24	0.66	0.	0.	0.	0.
time (sec)	N/A	0.532	0.178	0.006	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	126	320	0	0	167	0
normalized size	1	1.	0.32	0.81	0.	0.	0.42	0.
time (sec)	N/A	0.327	0.113	0.006	0.	0.	15.871	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	99	217	0	0	78	0
normalized size	1	1.	0.38	0.83	0.	0.	0.3	0.
time (sec)	N/A	0.096	0.042	0.004	0.	0.	8.421	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	732	732	432	564	0	0	0	0
normalized size	1	1.	0.59	0.77	0.	0.	0.	0.
time (sec)	N/A	0.781	0.805	0.023	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1494	1494	427	1384	0	0	0	0
normalized size	1	1.	0.29	0.93	0.	0.	0.	0.
time (sec)	N/A	2.007	1.626	0.033	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2452	2452	630	2326	0	0	0	0
normalized size	1	1.	0.26	0.95	0.	0.	0.	0.
time (sec)	N/A	3.971	3.033	0.035	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.219	0.396	0.066	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	94	81	0	0	0	0
normalized size	1	1.	1.96	1.69	0.	0.	0.	0.
time (sec)	N/A	0.029	0.136	0.028	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	755	755	4473	1708	0	0	0	0
normalized size	1	1.	5.92	2.26	0.	0.	0.	0.
time (sec)	N/A	1.357	6.506	0.021	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	674	1201	0	0	0	0
normalized size	1	1.	1.28	2.27	0.	0.	0.	0.
time (sec)	N/A	0.726	4.58	0.009	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	521	759	0	0	0	0
normalized size	1	1.	1.42	2.06	0.	0.	0.	0.
time (sec)	N/A	0.254	2.234	0.007	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0
normalized size	1	1.	1.07	1.28	0.	0.	0.	0.
time (sec)	N/A	0.081	0.265	0.006	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	298	359	0	0	0	0
normalized size	1	1.	0.68	0.82	0.	0.	0.	0.
time (sec)	N/A	0.409	0.513	0.029	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	782	782	1853	1495	0	0	0	0
normalized size	1	1.	2.37	1.91	0.	0.	0.	0.
time (sec)	N/A	1.467	3.822	0.027	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1125	1125	781	4476	0	0	0	0
normalized size	1	1.	0.69	3.98	0.	0.	0.	0.
time (sec)	N/A	3.594	6.17	0.031	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	859	859	5432	2445	0	0	0	0
normalized size	1	1.	6.32	2.85	0.	0.	0.	0.
time (sec)	N/A	1.38	6.674	0.034	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	633	766	1891	0	0	0	0
normalized size	1	1.01	1.22	3.01	0.	0.	0.	0.
time (sec)	N/A	0.584	4.764	0.008	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	481	481	0	1390	0	0	0	0
normalized size	1	1.	0.	2.89	0.	0.	0.	0.
time (sec)	N/A	0.387	0.	0.006	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	398	398	0	931	0	0	0	0
normalized size	1	1.	0.	2.34	0.	0.	0.	0.
time (sec)	N/A	0.195	0.	0.006	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	867	1045	1736	3241	0	0	0	0
normalized size	1	1.21	2.	3.74	0.	0.	0.	0.
time (sec)	N/A	1.424	5.243	0.026	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1301	2112	8031	8276	0	0	0	0
normalized size	1	1.62	6.17	6.36	0.	0.	0.	0.
time (sec)	N/A	3.386	8.007	0.041	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	310	365	0	0	0	0
normalized size	1	1.	1.14	1.34	0.	0.	0.	0.
time (sec)	N/A	0.136	0.537	0.056	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	312	369	0	0	0	0
normalized size	1	1.	1.15	1.36	0.	0.	0.	0.
time (sec)	N/A	0.187	0.443	0.042	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	74	93	0	0	0	0
normalized size	1	1.	0.7	0.88	0.	0.	0.	0.
time (sec)	N/A	0.105	0.183	0.028	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	218	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.463	0.242	0.042	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	125	296	0	744	0	0
normalized size	1	1.	1.18	2.79	0.	7.02	0.	0.
time (sec)	N/A	0.129	0.201	0.053	0.	2.419	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	441	238	0	1211	0	0
normalized size	1	1.	3.04	1.64	0.	8.35	0.	0.
time (sec)	N/A	0.207	0.661	0.033	0.	22.774	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	432	568	0	1364	0	0
normalized size	1	1.	1.81	2.38	0.	5.71	0.	0.
time (sec)	N/A	0.184	0.694	0.072	0.	22.672	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	214	442	0	1118	0	486
normalized size	1	1.	0.69	1.43	0.	3.62	0.	1.57
time (sec)	N/A	0.625	0.345	0.018	0.	2.051	0.	1.194

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	168	373	0	857	0	358
normalized size	1	1.	0.59	1.32	0.	3.03	0.	1.27
time (sec)	N/A	0.341	0.231	0.008	0.	1.924	0.	1.152

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	176	251	0	1798	0	0
normalized size	1	1.	0.62	0.88	0.	6.29	0.	0.
time (sec)	N/A	0.754	0.322	0.007	0.	7.154	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	171	289	0	1731	0	0
normalized size	1	1.	0.58	0.98	0.	5.89	0.	0.
time (sec)	N/A	0.767	0.261	0.011	0.	6.287	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	177	329	0	1766	0	500
normalized size	1	1.	0.61	1.14	0.	6.13	0.	1.74
time (sec)	N/A	0.784	0.359	0.01	0.	5.942	0.	1.236

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	189	412	0	1887	0	972
normalized size	1	1.	0.64	1.4	0.	6.42	0.	3.31
time (sec)	N/A	0.787	0.293	0.011	0.	6.503	0.	1.472

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [14] had the largest ratio of [0.3929]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	5	1.94	28	0.179
2	A	12	5	1.92	28	0.179
3	A	8	5	1.56	26	0.192
4	A	3	3	1.	19	0.158
5	A	3	3	1.	28	0.107
6	A	6	6	1.	28	0.214
7	A	7	6	1.	28	0.214
8	A	12	7	1.	28	0.25
9	A	10	6	1.	28	0.214
10	A	7	5	1.	26	0.192
11	A	4	4	1.	19	0.21
12	A	9	7	1.	28	0.25
13	A	15	10	1.	28	0.357
14	A	22	11	1.	28	0.393
15	A	6	3	1.	26	0.115
16	A	2	2	1.	27	0.074
17	A	6	4	1.	33	0.121
18	A	5	4	1.	33	0.121
19	A	4	4	1.	31	0.129
20	A	3	3	1.	24	0.125
21	A	3	3	1.	33	0.091
22	A	6	6	1.	33	0.182
23	A	7	6	1.	33	0.182
24	A	5	5	1.	33	0.152
25	A	4	4	1.01	33	0.121
26	A	4	4	1.	31	0.129
27	A	4	4	1.	24	0.167
28	A	9	7	1.21	33	0.212
29	A	15	10	1.62	33	0.303
30	A	1	1	1.	41	0.024
31	A	1	1	1.	41	0.024
32	A	4	4	1.	31	0.129
33	A	6	3	1.	31	0.097
34	A	11	7	1.	28	0.25
35	A	4	3	1.	30	0.1
36	A	1	1	1.	30	0.033
37	A	6	6	1.	38	0.158
38	A	6	6	1.	37	0.162
39	A	8	7	1.	40	0.175
40	A	8	7	1.	40	0.175
41	A	8	7	1.	40	0.175
42	A	8	7	1.	40	0.175

Chapter 3

Listing of integrals

3.1 $\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=453

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-63a^{3/2}\sqrt{c}e^2(Ae+3Bd)+25a^2Be^3+105\sqrt{a}c^{3/2}d^2(3Ae+Bd)\right.}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}$$

```
[Out] (e*(21*B*c*d^2 + 21*A*c*d*e - 5*a*B*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (e^2
*(3*B*d + A*e)*x^3*Sqrt[a + c*x^4])/(5*c) + (B*e^3*x^5*Sqrt[a + c*x^4])/(7*
c) + ((5*B*c*d^3 + 15*A*c*d^2*e - 9*a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x^4
])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(5*B*c*d^3 + 15*A*c*d^2*e
 - 9*a*B*d*e^2 - 3*a*A*e^3)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7
/4)*Sqrt[a + c*x^4]) + ((105*A*c^2*d^3 + 25*a^2*B*e^3 - 105*a*c*d*e*(B*d +
A*e) - 63*a^(3/2)*Sqrt[c]*e^2*(3*B*d + A*e) + 105*Sqrt[a]*c^(3/2)*d^2*(B*d
+ 3*A*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(210*a^(1/4)*c^(9/4)*Sqrt
[a + c*x^4])
```

Rubi [A]   time = 0.509473, antiderivative size = 878, normalized size of antiderivative = 1.94, number of steps used = 15, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1721, 220, 305, 1196, 321}

$$\frac{Be^3\sqrt{cx^4+ax^5}}{7c}+\frac{e^2(3Bd+Ae)\sqrt{cx^4+ax^3}}{5c}-\frac{5aBe^3\sqrt{cx^4+ax}}{21c^2}+\frac{de(Bd+Ae)\sqrt{cx^4+ax}}{c}-\frac{3ae^2(3Bd+Ae)\sqrt{cx^4+ax}}{5c^{3/2}\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + c*x^4],x]
```

```
[Out] (-5*a*B*e^3*x*Sqrt[a + c*x^4])/(21*c^2) + (d*e*(B*d + A*e)*x*Sqrt[a + c*x^4
])/c + (e^2*(3*B*d + A*e)*x^3*Sqrt[a + c*x^4])/(5*c) + (B*e^3*x^5*Sqrt[a +
c*x^4])/(7*c) - (3*a*e^2*(3*B*d + A*e)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[
a] + Sqrt[c]*x^2)) + (d^2*(B*d + 3*A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a
] + Sqrt[c]*x^2)) + (3*a^(5/4)*e^2*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) - (a^(1/4)*d^2*(B*d + 3*A*e)*(Sqr
```

```
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (A*d^3*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) + (
5*a^(7/4)*B*e^3*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(42*c^(9/4)*Sqrt[a
+ c*x^4]) - (a^(3/4)*d*e*(B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(2*c^(5/4)*Sqrt[a + c*x^4]) - (3*a^(5/4)*e^2*(3*B*d + A*e)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4]) + (a^(1/4)*d^2*(B*d
+ 3*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4]
)
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^3}{\sqrt{a + cx^4}}\, dx = \int \left(\frac{Ad^3}{\sqrt{a + cx^4}} + \frac{d^2(Bd + 3Ae)x^2}{\sqrt{a + cx^4}} + \frac{3de(Bd + Ae)x^4}{\sqrt{a + cx^4}} + \frac{e^2(3Bd + Ae)x^6}{\sqrt{a + cx^4}} + \frac{Be^3x^8}{\sqrt{a + cx^4}} \right) dx$$

$$= \left(Ad^3\right)\int \frac{1}{\sqrt{a + cx^4}}\, dx + \left(Be^3\right)\int \frac{x^8}{\sqrt{a + cx^4}}\, dx + (3de(Bd + Ae))\int \frac{x^4}{\sqrt{a + cx^4}}\, dx + \left(e^2(3\right.$$

$$= \frac{de(Bd + Ae)x\sqrt{a + cx^4}}{c} + \frac{e^2(3Bd + Ae)x^3\sqrt{a + cx^4}}{5c} + \frac{Be^3x^5\sqrt{a + cx^4}}{7c} + \frac{Ad^3\left(\sqrt{a} + \sqrt{cx}\right.}{}$$

$$= -\frac{5aBe^3x\sqrt{a + cx^4}}{21c^2} + \frac{de(Bd + Ae)x\sqrt{a + cx^4}}{c} + \frac{e^2(3Bd + Ae)x^3\sqrt{a + cx^4}}{5c} + \frac{Be^3x^5\sqrt{a + cx^4}}{7c}$$

$$= -\frac{5aBe^3x\sqrt{a + cx^4}}{21c^2} + \frac{de(Bd + Ae)x\sqrt{a + cx^4}}{c} + \frac{e^2(3Bd + Ae)x^3\sqrt{a + cx^4}}{5c} + \frac{Be^3x^5\sqrt{a + cx^4}}{7c}$$

Mathematica [C]   time = 0.281571, size = 217, normalized size = 0.48

$$\frac{7cx^3\sqrt{\frac{cx^4}{a} + 1}\ {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right)\left(-3aAe^3 - 9aBde^2 + 15Acd^2e + 5Bcd^3\right) + 5x\sqrt{\frac{cx^4}{a} + 1}\ {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right)\left(21Acd\left(cd^2\right.\right.}{105c^2\sqrt{a + cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*(a + c*x^4)*(-25*a*B*e^2 + 21*A*c*e*(5*d + e*x^2) + 3*B*c*(35*d^2 + 21
*d*e*x^2 + 5*e^2*x^4)) + 5*(21*A*c*d*(c*d^2 - a*e^2) + a*B*e*(-21*c*d^2 + 5
*a*e^2))*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a
)] + 7*c*(5*B*c*d^3 + 15*A*c*d^2*e - 9*a*B*d*e^2 - 3*a*A*e^3)*x^3*Sqrt[1 +
(c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(105*c^2*Sqrt[a
+ c*x^4])
```

Maple [C]   time = 0.054, size = 533, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] B*e^3*(1/7/c*x^5*(c*x^4+a)^(1/2)-5/21*a/c^2*x*(c*x^4+a)^(1/2)+5/21*a^2/c^2/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+(A
*e^3+3*B*d*e^2)*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)
*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*
(I/a^(1/2)*c^(1/2))^(1/2),I)))+(3*A*d*e^2+3*B*d^2*e)*(1/3/c*x*(c*x^4+a)^(1/
2)-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(
1/2),I))+I*(3*A*d^2*e+B*d^3)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)
```

```
*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)
*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^
(1/2),I))+A*d^3/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/
2))^(1/2),I)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out]  integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Be^3x^8 + \left(3\,Bde^2 + Ae^3\right)x^6 + 3\left(Bd^2e + Ade^2\right)x^4 + Ad^3 + \left(Bd^3 + 3\,Ad^2e\right)x^2}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out]  integral((B*e^3*x^8 + (3*B*d*e^2 + A*e^3)*x^6 + 3*(B*d^2*e + A*d*e^2)*x^4 +
  A*d^3 + (B*d^3 + 3*A*d^2*e)*x^2)/sqrt(c*x^4 + a), x)
```

Sympy [C]   time = 6.84416, size = 364, normalized size = 0.8

$$\frac{Ad^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3Ad^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3Ade^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Ae^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{cx}{}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out]  A*d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + 3*A*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,),
c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*A*d*e**2*x**5*gamma(5/
4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4
)) + A*e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*p
i)/a)/(4*sqrt(a)*gamma(11/4)) + B*d**3*x**3*gamma(3/4)*hyper((1/2, 3/4), (7
/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*B*d**2*e*x**5*ga
mma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gam
ma(9/4)) + 3*B*d*e**2*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4)) + B*e**3*x**9*gamma(9/4)*hyper((1/2
```

, 9/4), (13/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + a), x)

3.2 $\quad \int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)^2}{\sqrt{a+cx^4}}\,dx$

<u>**Optimal**</u>. Leaf size=367

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-9a^{3/2}Be^2+15\sqrt{a}cd(2Ae+Bd)-5a\sqrt{c}e(Ae+2Bd)+15Ac^{3/2}d^2\right.}{30\sqrt[4]{a}c^{7/4}\sqrt{a+cx^4}}$$

```
[Out] (e*(2*B*d + A*e)*x*Sqrt[a + c*x^4])/(3*c) + (B*e^2*x^3*Sqrt[a + c*x^4])/(5*
c) + ((5*B*c*d^2 + 10*A*c*d*e - 3*a*B*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(5*B*c*d^2 + 10*A*c*d*e - 3*a*B*e^2)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) + ((15*A*c^
(3/2)*d^2 - 9*a^(3/2)*B*e^2 - 5*a*Sqrt[c]*e*(2*B*d + A*e) + 15*Sqrt[a]*c*d*
(B*d + 2*A*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(30*a^(1/4)*c^(7/4)*
Sqrt[a + c*x^4])
```

<u>**Rubi [A]**</u>   time = 0.395883, antiderivative size = 706, normalized size of antiderivative = 1.92, number of steps used = 12, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1721, 220, 305, 1196, 321}

$$-\frac{a^{3/4}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Ae+2Bd)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}-\frac{3a^{5/4}Be^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*(2*B*d + A*e)*x*Sqrt[a + c*x^4])/(3*c) + (B*e^2*x^3*Sqrt[a + c*x^4])/(5*
c) - (3*a*B*e^2*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (d
*(B*d + 2*A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (3*a^
(5/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x
^4]) - (a^(1/4)*d*(B*d + 2*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^
(3/4)*Sqrt[a + c*x^4]) + (A*d^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*
a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) - (3*a^(5/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4]) - (a^(3/4)*e*(2*B*d + A*e)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4]) + (a^(1/4
)*d*(B*d + 2*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a
 + c*x^4])
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
```

```
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(A + Bx^2)(d + ex^2)^2}{\sqrt{a + cx^4}}\, dx = \int \left(\frac{Ad^2}{\sqrt{a + cx^4}} + \frac{d(Bd + 2Ae)x^2}{\sqrt{a + cx^4}} + \frac{e(2Bd + Ae)x^4}{\sqrt{a + cx^4}} + \frac{Be^2x^6}{\sqrt{a + cx^4}} \right) dx$$

$$= (Ad^2) \int \frac{1}{\sqrt{a + cx^4}}\, dx + (Be^2) \int \frac{x^6}{\sqrt{a + cx^4}}\, dx + (e(2Bd + Ae)) \int \frac{x^4}{\sqrt{a + cx^4}}\, dx + (d(Bd$$

$$= \frac{e(2Bd + Ae)x\sqrt{a + cx^4}}{3c} + \frac{Be^2x^3\sqrt{a + cx^4}}{5c} + \frac{Ad^2\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a + cx^4}{(\sqrt{a} + \sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a + cx^4}}$$

$$= \frac{e(2Bd + Ae)x\sqrt{a + cx^4}}{3c} + \frac{Be^2x^3\sqrt{a + cx^4}}{5c} + \frac{d(Bd + 2Ae)x\sqrt{a + cx^4}}{\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}d(Bd + 2Ae)\left(}{}$$

$$= \frac{e(2Bd + Ae)x\sqrt{a + cx^4}}{3c} + \frac{Be^2x^3\sqrt{a + cx^4}}{5c} - \frac{3aBe^2x\sqrt{a + cx^4}}{5c^{3/2}\left(\sqrt{a} + \sqrt{c}x^2\right)} + \frac{d(Bd + 2Ae)x\sqrt{a + cx}}{\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)}$$

Mathematica [C]   time = 0.170828, size = 159, normalized size = 0.43

$$\frac{x^3\sqrt{\frac{cx^4}{a} + 1}\,_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right)\left(-3aBe^2 + 10Acde + 5Bcd^2\right) - 5x\sqrt{\frac{cx^4}{a} + 1}\,_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right)\left(aAe^2 + 2aBde - 3Acd^2\right)}{15c\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*(10*B*d + 5*A*e + 3*B*e*x^2)*(a + c*x^4) - 5*(-3*A*c*d^2 + 2*a*B*d*e +
 a*A*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/
a)] + (5*B*c*d^2 + 10*A*c*d*e - 3*a*B*e^2)*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(15*c*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.008, size = 403, normalized size = 1.1

$$
Be^2\left[\frac{x^3}{5c}\sqrt{cx^4+a}-\frac{3\,i}{5}a^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\right]c^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{c}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] B*e^2*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^
(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)))+(A*e^2+2*B*d*e)*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I/a^
(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+I*(2*A*
d*e+B*d^2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+A*d^2/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(Bx^2+A\right)\left(ex^2+d\right)^2}{\sqrt{cx^4+a}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$
\text{integral}\left(\frac{B e^2 x^6 + \left(2\,Bde + Ae^2\right)x^4 + Ad^2 + \left(Bd^2 + 2\,Ade\right)x^2}{\sqrt{cx^4+a}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^6 + (2*B*d*e + A*e^2)*x^4 + A*d^2 + (B*d^2 + 2*A*d*e)*x^2
)/sqrt(c*x^4 + a), x)
```

Sympy [C]   time = 4.88839, size = 262, normalized size = 0.71

$$\frac{Ad^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Adex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Ae^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Bd^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] A*d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + A*d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**
4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + A*e**2*x**5*gamma(5/4)*hyper(
(1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + B*d*
*2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(7/4)) + B*d*e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4
*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**2*x**7*gamma(7/4)*hyper((
1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2+A\right)\left(ex^2+d\right)^2}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

3.3 $\int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=277

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\frac{3Acd-aBe}{\sqrt{a}}+3\sqrt{c}(Ae+Bd)\right)}{6c^{5/4}\sqrt{a+cx^4}}\quad \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(}{c^{3/4}\sqrt{a}}$$

```
[Out] (B*e*x*Sqrt[a + c*x^4])/(3*c) + ((B*d + A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(3*Sqrt[c]*(B*d + A*e) + (3
*A*c*d - a*B*e)/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)
*Sqrt[a + c*x^4])
```

Rubi [A]   time = 0.224547, antiderivative size = 432, normalized size of antiderivative = 1.56, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1721, 220, 305, 1196, 321}

$$-\frac{a^{3/4}Be\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Ae+Bd)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2))/Sqrt[a + c*x^4],x]
```

```
[Out] (B*e*x*Sqrt[a + c*x^4])/(3*c) + ((B*d + A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (A*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) - (a^(3/4)*B*e*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(B*d + A*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

<u>Rule 1196</u>

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

<u>Rule 321</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rubi steps</u>

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)}{\sqrt{a + cx^4}}\,dx = \int \left(\frac{Ad}{\sqrt{a + cx^4}} + \frac{(Bd + Ae)x^2}{\sqrt{a + cx^4}} + \frac{Bex^4}{\sqrt{a + cx^4}}\right)dx$$

$$= (Ad)\int \frac{1}{\sqrt{a + cx^4}}\,dx + (Be)\int \frac{x^4}{\sqrt{a + cx^4}}\,dx + (Bd + Ae)\int \frac{x^2}{\sqrt{a + cx^4}}\,dx$$

$$= \frac{Bex\sqrt{a + cx^4}}{3c} + \frac{Ad\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a + cx^4}} - \frac{(aBe)\int \frac{1}{\sqrt{a + cx^4}}\,dx}{3c}$$

$$= \frac{Bex\sqrt{a + cx^4}}{3c} + \frac{(Bd + Ae)x\sqrt{a + cx^4}}{\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(Bd + Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}}E\left(2\tan\right.}{c^{3/4}\sqrt{a + cx^4}}$$

Mathematica [C]   time = 0.0990607, size = 121, normalized size = 0.44

$$\frac{cx^3\sqrt{\frac{cx^4}{a} + 1}\,{}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right)(Ae + Bd) + x\sqrt{\frac{cx^4}{a} + 1}\,{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right)(3Acd - aBe) + Bex\left(a + cx^4\right)}{3c\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/Sqrt[a + c*x^4],x]
```

```
[Out] (B*e*x*(a + c*x^4) + (3*A*c*d - a*B*e)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + c*(B*d + A*e)*x^3*Sqrt[1 + (c*x^4)/a]*Hy
pergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(3*c*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.004, size = 269, normalized size = 1.

$$Be\left(\frac{x}{3c}\sqrt{cx^4 + a} - \frac{a}{3c}\sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4 + a}}\right) + i(Ae + Bd)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] B*e*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+I*(A*e+B*d)*a^(1/2)/(I/a^(1/2)*c^(1/2))^
(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I
/a^(1/2)*c^(1/2))^(1/2),I))+A*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bex^4 + (Bd + Ae)x^2 + Ad}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)/sqrt(c*x^4 + a), x)
```

Sympy [C]   time = 3.29101, size = 167, normalized size = 0.6

$$\frac{Adx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] A*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper((1/2, 3/
4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*e*x**5*gam
ma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
```

```
a(9/4))
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + a), x)
```

3.4 $\int \dfrac{A+Bx^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right)}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (B*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*B*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(B +
 (A*Sqrt[c])/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rubi [A]   time = 0.0656975, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*B*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(B +
 (A*Sqrt[c])/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A + Bx^2}{\sqrt{a + cx^4}}\,dx = \left(A + \frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a + cx^4}}\,dx - \frac{\left(\sqrt{a}B\right)\int \frac{1 - \frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a + cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{Bx\sqrt{a + cx^4}}{\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a + cx^4}} + \frac{\left(\sqrt{a}B + A\sqrt{c}\right)\left(\sqrt{a} + \sqrt{c}x}{2\sqrt[4]{}}$$

Mathematica [C]   time = 0.0340073, size = 77, normalized size = 0.34

$$\frac{\sqrt{\frac{cx^4}{a} + 1}\left(3Ax\,_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right) + Bx^3\,_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{cx^4}{a}\right)\right)}{3\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(3*A*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ B*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^4]
)
```

Maple [C]   time = 0.005, size = 169, normalized size = 0.8

$$iB\sqrt{a}\sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4 + a}}\frac{1}{\sqrt{c}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+a)^(1/2),x)
```

```
[Out] I*B*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+A/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/sqrt(c*x^4 + a), x)
```

Sympy [C]   time = 1.59122, size = 78, normalized size = 0.35

$$\frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+a)**(1/2),x)
```

```
[Out] A*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_pol
ar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + a), x)
```

3.5 $\quad \int \dfrac{A+Bx^2}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=369

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B-A\sqrt{c}\right)\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(l}{4\sqrt[4]{c}de\sqrt{a+cx}}$$

```
[Out] -((B*d - A*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^
4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) - ((Sqrt[a]*B - A*Sqrt[c])*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[
a]*e)*Sqrt[a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
Pi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(4*c^(1/4)*d*e*(c*d^2 - a*e^2)*Sqrt[a + c*x^4])
```

Rubi [A]   time = 0.360138, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1709, 220, 1707}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{cde}};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{cde}\sqrt{a+cx^4}\left(cd^2-ae^2\right)}-\frac{(Bd-Ae)\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] -((B*d - A*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^
4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) - ((Sqrt[a]*B - A*Sqrt[c])*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[
a]*e)*Sqrt[a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
Pi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(4*c^(1/4)*d*e*(c*d^2 - a*e^2)*Sqrt[a + c*x^4])
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)\sqrt{a + cx^4}}\, dx = -\frac{\left(\sqrt{a}B - A\sqrt{c}\right)\int \frac{1}{\sqrt{a+cx^4}}\, dx}{\sqrt{c}d - \sqrt{a}e} + \frac{\left(\sqrt{a}(Bd - Ae)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+cx^4}}\, dx}{\sqrt{c}d - \sqrt{a}e}$$

$$= -\frac{(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2 + ae^2}} - \frac{\left(\sqrt{a}B - A\sqrt{c}\right)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right|}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d - \sqrt{a}e\right)\sqrt{a + cx^4}}$$

Mathematica [C]   time = 0.304231, size = 138, normalized size = 0.37

$$-\frac{i\sqrt{\frac{cx^4}{a} + 1}\left(Bd\,\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right), -1\right) + (Ae - Bd)\Pi\left(-\frac{i\sqrt{ae}}{\sqrt{cd}}; i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\Big| - 1\right)\right)}{de\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*(B*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1] + (-(B*d) + A*e)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e*Sqrt[
a + c*x^4])
```

Maple [C]   time = 0.033, size = 192, normalized size = 0.5

$$\frac{B}{e}\sqrt{1 - ix^2\sqrt{c}\sqrt{\frac{1}{\sqrt{a}}}}\sqrt{1 + ix^2\sqrt{c}\sqrt{\frac{1}{\sqrt{a}}}}\,\text{EllipticF}\left(x\sqrt{i\sqrt{c}\sqrt{\frac{1}{\sqrt{a}}}}, i\right)\frac{1}{\sqrt{i\sqrt{c}\sqrt{\frac{1}{\sqrt{a}}}}}\frac{1}{\sqrt{cx^4 + a}} + \frac{Ae - Bd}{ed}\sqrt{1 - ix^2\sqrt{c}\sqrt{\frac{1}{\sqrt{a}}}}\sqrt{1 + ix^2\sqrt{c}\sqrt{\frac{1}{\sqrt{a}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] B/e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+(A*e-B*d)/e/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
```

2))^(1/2))

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\sqrt{a + cx^4}\left(d + ex^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+a)**(1/2),x)

[Out] Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)), x)

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)

3.6 $\int \dfrac{A+Bx^2}{\left(d+ex^2\right)^2 \sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=641

$$\dfrac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}+\dfrac{\sqrt{c}x\sqrt{a+cx^4}(Bd-Ae)}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}-\dfrac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-$$

```
[Out] (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqr
t[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*
x^2)) - ((B*c*d^3 - 3*A*c*d^2*e - a*B*d*e^2 - a*A*e^3)*ArcTan[(Sqrt[c*d^2 +
 a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(4*d^(3/2)*Sqrt[e]*(c*d^2 +
a*e^2)^(3/2)) - (a^(1/4)*c^(1/4)*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (A*c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x
^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*c*d^3 - 3*A*c*d^2*e - a*B*d*e^2 - a*A*e^
3)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^
2 + a*e^2)*Sqrt[a + c*x^4])
```

Rubi [A]   time = 1.18937, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1697, 1715, 1196, 1709, 220, 1707}

$$\dfrac{\sqrt{c}x\sqrt{a+cx^4}(Bd-Ae)}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}-\dfrac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-\dfrac{\left(-aAe^3-aBde^2-3Acd^2e+Bcd^3\right)\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\sqrt{e}\left(ae^2+cd^2\right)^{3/2}}-\dfrac{\sqrt[4]{a}\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqr
t[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*
x^2)) - ((B*c*d^3 - 3*A*c*d^2*e - a*B*d*e^2 - a*A*e^3)*ArcTan[(Sqrt[c*d^2 +
 a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(4*d^(3/2)*Sqrt[e]*(c*d^2 +
a*e^2)^(3/2)) - (a^(1/4)*c^(1/4)*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (A*c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x
^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*c*d^3 - 3*A*c*d^2*e - a*B*d*e^2 - a*A*e^
3)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^
2 + a*e^2)*Sqrt[a + c*x^4])
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
```

```
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)])*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^2 \sqrt{a + cx^4}}\, dx = -\frac{e(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)\left(d + ex^2\right)} - \frac{\int \frac{-2Acd^2 - aBde - aAe^2 - 2cd(Bd - Ae)x^2 - ce(Bd - Ae)x^4}{\left(d+ex^2\right)\sqrt{a+cx^4}}\, dx}{2d\left(cd^2 + ae^2\right)}$$

$$= -\frac{e(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)\left(d + ex^2\right)} - \frac{\int \frac{-\sqrt{a}c^{3/2}de(Bd-Ae)+ce\left(-2Acd^2-aBde-aAe^2\right)+\left(-2c^2de(Bd-Ae)+ce(Bd-Ae)\left(cd-\sqrt{}\right)\right)}{\left(d+ex^2\right)\sqrt{a+cx^4}}}{2cde\left(cd^2 + ae^2\right)}$$

$$= \frac{\sqrt{c}(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)\left(d + ex^2\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{}}{2d\left(cd^2 + ae^2\right)}$$

$$= \frac{\sqrt{c}(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)\left(d + ex^2\right)} - \frac{\left(Bcd^3 - 3Acd^2e - aBde^2 - aAe^3\right)}{4d^{3/2}\sqrt{e}\left(cd^2 + ae^2\right)}$$

Mathematica [C]   time = 1.23753, size = 297, normalized size = 0.46

$$\frac{dex\left(a+cx^4\right)(Ae-Bd)}{\left(d+ex^2\right)\left(ae^2+cd^2\right)} - \frac{i\sqrt{\frac{cx^4}{a}+1}\left(\sqrt{cd}\left(\sqrt{cd}-i\sqrt{ae}\right)(Bd-Ae)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right),-1\right)+\left(aAe^3+aBde^2+3Acd^2e-Bcd^3\right)\Pi\left(-\frac{i\sqrt{ae}}{\sqrt{cd}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\sqrt{a}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(ae^3+cd^2e\right)}}{2d^2\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] ((d*e*(-(B*d) + A*e)*x*(a + c*x^4))/((c*d^2 + a*e^2)*(d + e*x^2)) - (I*Sqrt
[1 + (c*x^4)/a]*(I*Sqrt[a]*Sqrt[c]*d*e*(B*d - A*e)*EllipticE[I*ArcSinh[Sqrt
[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(B*d -
A*e)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (-(B*c*d^3) +
3*A*c*d^2*e + a*B*d*e^2 + a*A*e^3)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d),
 I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*(
c*d^2*e + a*e^3)))/(2*d^2*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.025, size = 679, normalized size = 1.1

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] (A*e-B*d)/e*(1/2*e^2/(a*e^2+c*d^2)/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-1/2*c/(a*e
^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1
/2),I)-1/2*I*c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/(a*e^2+c*d^2)/d
*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/
2),I)+1/2/(a*e^2+c*d^2)/d^2*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
```

```
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x
*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)
/(I/a^(1/2)*c^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2
)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c)+B/e/d/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a
^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\sqrt{a + cx^4}\left(d + ex^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.7 $\int \dfrac{A+Bx^2}{\left(d+ex^2\right)^3 \sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=875

$$\frac{\sqrt{c}\left(5Bcd^3 - 9Aced^2 - aBe^2d - 3aAe^3\right)\sqrt{cx^4+ax}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)} - \frac{e\left(5Bcd^3 - 9Aced^2 - aBe^2d - 3aAe^3\right)\sqrt{cx^4+ax}}{8d^2\left(cd^2+ae^2\right)^2\left(ex^2+d\right)} - \frac{e(Bd-Ae)\sqrt{}}{4d\left(cd^2+ae^2\right)}$$

```
[Out] (Sqrt[c]*(5*B*c*d^3 - 9*A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x^4
])/(8*d^2*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqr
t[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d + e*x^2)^2) - (e*(5*B*c*d^3 - 9*A*c*d
^2*e - a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2*(
d + e*x^2)) + ((3*A*e*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - B*(3*c^2*d^5
- 10*a*c*d^3*e^2 - a^2*d*e^4))*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt
[e]*Sqrt[a + c*x^4])])/(16*d^(5/2)*Sqrt[e]*(c*d^2 + a*e^2)^(5/2)) - (a^(1/4
)*c^(1/4)*(5*B*c*d^3 - 9*A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/
4)*(4*A*c*d^2 + Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(B*d + 3*A*e))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
^2 + a*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*A*e*(5*c^2*d^4 +
 2*a*c*d^2*e^2 + a^2*e^4) - B*(3*c^2*d^5 - 10*a*c*d^3*e^2 - a^2*d*e^4))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi
[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a
^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a
*e^2)^2*Sqrt[a + c*x^4])
```

Rubi [A]   time = 1.78415, antiderivative size = 875, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1697, 1715, 1196, 1709, 220, 1707}

$$\frac{\sqrt{c}\left(5Bcd^3 - 9Aced^2 - aBe^2d - 3aAe^3\right)\sqrt{cx^4+ax}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)} - \frac{e\left(5Bcd^3 - 9Aced^2 - aBe^2d - 3aAe^3\right)\sqrt{cx^4+ax}}{8d^2\left(cd^2+ae^2\right)^2\left(ex^2+d\right)} - \frac{e(Bd-Ae)\sqrt{}}{4d\left(cd^2+ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[c]*(5*B*c*d^3 - 9*A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x^4
])/(8*d^2*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqr
t[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d + e*x^2)^2) - (e*(5*B*c*d^3 - 9*A*c*d
^2*e - a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2*(
d + e*x^2)) + ((3*A*e*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - B*(3*c^2*d^5
- 10*a*c*d^3*e^2 - a^2*d*e^4))*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt
[e]*Sqrt[a + c*x^4])])/(16*d^(5/2)*Sqrt[e]*(c*d^2 + a*e^2)^(5/2)) - (a^(1/4
)*c^(1/4)*(5*B*c*d^3 - 9*A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/
4)*(4*A*c*d^2 + Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(B*d + 3*A*e))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
```

```
^2 + a*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*A*e*(5*c^2*d^4 +
 2*a*c*d^2*e^2 + a^2*e^4) - B*(3*c^2*d^5 - 10*a*c*d^3*e^2 - a^2*d*e^4))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi
[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a
^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a
*e^2)^2*Sqrt[a + c*x^4])
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[(((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^3 \sqrt{a + cx^4}}\, dx = -\frac{e(Bd - Ae)x\sqrt{a + cx^4}}{4d\left(cd^2 + ae^2\right)\left(d + ex^2\right)^2} - \frac{\int \frac{-4Acd^2 - aBde - 3aAe^2 - 4cd(Bd - Ae)x^2 + ce(Bd - Ae)x^4}{\left(d + ex^2\right)^2 \sqrt{a + cx^4}}\, dx}{4d\left(cd^2 + ae^2\right)}$$

$$= -\frac{e(Bd - Ae)x\sqrt{a + cx^4}}{4d\left(cd^2 + ae^2\right)\left(d + ex^2\right)^2} - \frac{e\left(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3\right)x\sqrt{a + cx^4}}{8d^2\left(cd^2 + ae^2\right)^2\left(d + ex^2\right)} + \int \frac{aBde\left(7}{}$$

$$= -\frac{e(Bd - Ae)x\sqrt{a + cx^4}}{4d\left(cd^2 + ae^2\right)\left(d + ex^2\right)^2} - \frac{e\left(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3\right)x\sqrt{a + cx^4}}{8d^2\left(cd^2 + ae^2\right)^2\left(d + ex^2\right)} + \int \frac{\sqrt{a}c^{3/2}}{}$$

$$= \frac{\sqrt{c}\left(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3\right)x\sqrt{a + cx^4}}{8d^2\left(cd^2 + ae^2\right)^2\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(Bd - Ae)x\sqrt{a + cx^4}}{4d\left(cd^2 + ae^2\right)\left(d + ex^2\right)^2} - \frac{e\left(5Bcd}{}$$

$$= \frac{\sqrt{c}\left(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3\right)x\sqrt{a + cx^4}}{8d^2\left(cd^2 + ae^2\right)^2\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(Bd - Ae)x\sqrt{a + cx^4}}{4d\left(cd^2 + ae^2\right)\left(d + ex^2\right)^2} - \frac{e\left(5Bcd}{}$$

Mathematica [C]   time = 2.29221, size = 453, normalized size = 0.52

$$-\frac{de^2x\left(a + cx^4\right)\left(\left(d + ex^2\right)\left(-3aAe^3 - aBde^2 - 9Acd^2e + 5Bcd^3\right) + 2d\left(ae^2 + cd^2\right)(Bd - Ae)\right)}{\left(d + ex^2\right)^2} - \frac{i\sqrt{\frac{cx^4}{a} + 1}\left(\sqrt{cd}\left(\sqrt{c}d - i\sqrt{a}e\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right), -1\right)\left(Ae\left(2i\sqrt{a}\sqrt{c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-((d*e^2*x*(a + c*x^4)*(2*d*(B*d - A*e)*(c*d^2 + a*e^2) + (5*B*c*d^3 - 9*A
*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*(d + e*x^2)))/(d + e*x^2)^2) - (I*Sqrt[1
+ (c*x^4)/a]*((-I)*Sqrt[a]*Sqrt[c]*d*e*(-5*B*c*d^3 + 9*A*c*d^2*e + a*B*d*e^
2 + 3*a*A*e^3)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt
[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(A*e*(-7*c*d^2 + (2*I)*Sqrt[a]*Sqrt[c]*d*e
- 3*a*e^2) + B*d*(3*c*d^2 - (2*I)*Sqrt[a]*Sqrt[c]*d*e - a*e^2))*EllipticF[I
*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (3*A*e*(5*c^2*d^4 + 2*a*c*d^2*
e^2 + a^2*e^4) + B*(-3*c^2*d^5 + 10*a*c*d^3*e^2 + a^2*d*e^4))*EllipticPi[((
-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/S
qrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*e*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.027, size = 1591, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x)`

[Out] `B/e*(1/2*e^2/(a*e^2+c*d^2)/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^2`
`)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^`
`(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1`
`/2*I*c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/`
`2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip`
`ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)`
`/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(`
`1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/`
`2/(a*e^2+c*d^2)/d^2*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)`
`^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1`
`/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1`
`/2)*c^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1`
`/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elli`
`pticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2`
`))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c)+(A*e-B*d)/e*(1/4*e^2/(a*e^2+c*d^2)/d`
`*x*(c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)/(a*e^2+c*d^2)^2/d^2*`
`x*(c*x^4+a)^(1/2)/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/`
`2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a`
`)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*a*e^2-7/8*c^2*d/(a*e^2+c*d`
`^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2`
`)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),`
`I)-3/8*I*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*`
`(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(`
`1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+9/8*I*c^(3/2)*e/(a*e^2+c*d^2)`
`^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a`
`^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(`
`1/2),I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(`
`1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1`
`/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8*I*c^(1/2)*e^3/(a*e^2+c*d^2`
`)^2/d^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(`
`1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/`
`2))^(1/2),I)+3/8/(a*e^2+c*d^2)^2/d^3*e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(`
`1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ell`
`ipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/`
`2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4/(a*e^2+c*d^2)^2*e^2/d/(I/a^(1/`
`2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)`
`^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(`
`1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a*c+15/8/(a*`
`e^2+c*d^2)^2*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1`
`+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/`
`2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/`
`2))^(1/2))*c^2)`

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)`

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out]  Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\sqrt{a + cx^4}\left(d + ex^2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out]  Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out]  integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

3.8 $\int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)^3}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=912

<div align="center">result too large to display</div>

```
[Out] (x*(A*c*d*(c*d^2 - 3*a*e^2) - a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 + 3*A*c*
d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x^2))/(2*a*c^2*Sqrt[a + c*x^4]) + (B*e^3*x*S
qrt[a + c*x^4])/(3*c^2) + (e^2*(3*B*d + A*e)*x*Sqrt[a + c*x^4])/(c^(3/2)*(S
qrt[a] + Sqrt[c]*x^2)) - ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x
*Sqrt[a + c*x^4])/(2*a*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e^2*(3*B
*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(7/4)*Sqrt[a + c*x^4])
 + ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4]) - (a^(3/4)*B*e^3*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[a + c*x^4]) + (a^(1/4)*e
^2*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[a
+ c*x^4]) + (e*(3*B*c*d^2 + 3*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4]) + ((A*c^2*d^3 + a^2*B*e^3
 - 3*a*c*d*e*(B*d + A*e) + a^(3/2)*Sqrt[c]*e^2*(3*B*d + A*e) - Sqrt[a]*c^(3
/2)*d^2*(B*d + 3*A*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c
^(9/4)*Sqrt[a + c*x^4])
```

Rubi [A]   time = 0.72556, antiderivative size = 912, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1721, 1179, 1198, 220, 1196, 305, 321}

$$-\frac{a^{3/4}B\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^3}{6c^{9/4}\sqrt{cx^4+a}}+\frac{Bx\sqrt{cx^4+a}e^3}{3c^2}-\frac{\sqrt[4]{a}(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\right.}{c^{7/4}\sqrt{cx^4+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*c*d*(c*d^2 - 3*a*e^2) - a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 + 3*A*c*
d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x^2))/(2*a*c^2*Sqrt[a + c*x^4]) + (B*e^3*x*S
qrt[a + c*x^4])/(3*c^2) + (e^2*(3*B*d + A*e)*x*Sqrt[a + c*x^4])/(c^(3/2)*(S
qrt[a] + Sqrt[c]*x^2)) - ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x
*Sqrt[a + c*x^4])/(2*a*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e^2*(3*B
*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(7/4)*Sqrt[a + c*x^4])
 + ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4]) - (a^(3/4)*B*e^3*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[a + c*x^4]) + (a^(1/4)*e
^2*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[a
```

```
+ c*x^4]) + (e*(3*B*c*d^2 + 3*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4]) + ((A*c^2*d^3 + a^2*B*e^3
 - 3*a*c*d*e*(B*d + A*e) + a^(3/2)*Sqrt[c]*e^2*(3*B*d + A*e) - Sqrt[a]*c^(3
/2)*d^2*(B*d + 3*A*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c
^(9/4)*Sqrt[a + c*x^4])
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^3}{\left(a + cx^4\right)^{3/2}}\,dx = \int \left(\frac{Acd\left(cd^2 - 3ae^2\right) - aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 + 3Acd^2e - 3aBde^2 - aAe^3\right)x^2}{c^2\left(a + cx^4\right)^{3/2}} + \frac{e\left(3Bc\right.}{}\right.$$

$$= \frac{\int \frac{Acd\left(cd^2 - 3ae^2\right) - aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 + 3Acd^2e - 3aBde^2 - aAe^3\right)x^2}{\left(a + cx^4\right)^{3/2}}\,dx}{c^2} + \frac{\left(Be^3\right)\int \frac{x^4}{\sqrt{a + cx^4}}\,dx}{c} + \frac{\left(e^2(3Bd + \right.}{}$$

$$= \frac{x\left(Acd\left(cd^2 - 3ae^2\right) - aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 + 3Acd^2e - 3aBde^2 - aAe^3\right)x^2\right)}{2ac^2\sqrt{a + cx^4}} + \frac{Be^3x\sqrt{}}{3}$$

$$= \frac{x\left(Acd\left(cd^2 - 3ae^2\right) - aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 + 3Acd^2e - 3aBde^2 - aAe^3\right)x^2\right)}{2ac^2\sqrt{a + cx^4}} + \frac{Be^3x\sqrt{}}{3}$$

$$= \frac{x\left(Acd\left(cd^2 - 3ae^2\right) - aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 + 3Acd^2e - 3aBde^2 - aAe^3\right)x^2\right)}{2ac^2\sqrt{a + cx^4}} + \frac{Be^3x\sqrt{}}{3}$$

Mathematica [C]   time = 0.286017, size = 222, normalized size = 0.24

$$\frac{2cx^3\sqrt{\frac{cx^4}{a} + 1}\,{}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{cx^4}{a}\right)\left(-3aAe^3 - 9aBde^2 + 3Acd^2e + Bcd^3\right) + x\sqrt{\frac{cx^4}{a} + 1}\,{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right)\left(3Acd\left(3ae^2 + cd^2\right)\right.}{6ac^2\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x]
```

```
[Out] (3*A*c*x*(c*d^3 + a*e^2*(-3*d + 2*e*x^2)) + a*B*e*x*(5*a*e^2 + c*(-9*d^2 +
18*d*e*x^2 + 2*e^2*x^4)) + (a*B*e*(9*c*d^2 - 5*a*e^2) + 3*A*c*d*(c*d^2 + 3*
a*e^2))*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)
] + 2*c*(B*c*d^3 + 3*A*c*d^2*e - 9*a*B*d*e^2 - 3*a*A*e^3)*x^3*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c^2*Sqrt[a + c*
x^4])
```

Maple [C]   time = 0.036, size = 588, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x)
```

```
[Out] B*e^3*(1/2/c^2*a*x/((x^4+a/c)*c)^(1/2)+1/3/c^2*x*(c*x^4+a)^(1/2)-5/6*a/c^2/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+(A
*e^3+3*B*d*e^2)*(-1/2/c*x^3/((x^4+a/c)*c)^(1/2)+3/2*I/c^(3/2)*a^(1/2)/(I/a^
(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Ellipti
cE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+(3*A*d*e^2+3*B*d^2*e)*(-1/2/c*x/((x^4+a
/c)*c)^(1/2)+1/2/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^
```

```
(1/2))^(1/2),I))+(3*A*d^2*e+B*d^3)*(1/2/a*x^3/((x^4+a/c)*c)^(1/2)-1/2*I/a^(
1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+A*d^3*(1/2/a*x/((x^4+
a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c
^(1/2))^(1/2),I))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\left(cx^4 + a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Be^3x^8 + \left(3\,Bde^2 + Ae^3\right)x^6 + 3\left(Bd^2e + Ade^2\right)x^4 + Ad^3 + \left(Bd^3 + 3\,Ad^2e\right)x^2\right)\sqrt{cx^4 + a}}{c^2x^8 + 2\,acx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^3*x^8 + (3*B*d*e^2 + A*e^3)*x^6 + 3*(B*d^2*e + A*d*e^2)*x^4 +
 A*d^3 + (B*d^3 + 3*A*d^2*e)*x^2)*sqrt(c*x^4 + a)/(c^2*x^8 + 2*a*c*x^4 + a^
2), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^3}{\left(a + cx^4\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)*(d + e*x**2)**3/(a + c*x**4)**(3/2), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\left(cx^4 + a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + a)^(3/2), x)
```

3.9 $\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)^2}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=694

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\frac{\sqrt{c}\left(-aAe^2-2aBde+Acd^2\right)}{\sqrt{a}}-aBe^2+2Acde+Bcd^2\right)}{4a^{3/4}c^{7/4}\sqrt{a+cx^4}}+\frac{e\left(\sqrt{a}+\sqrt{}\right.}{}$$

```
[Out] (x*(A*c*d^2 - 2*a*B*d*e - a*A*e^2 + (B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x^2))/(
2*a*c*Sqrt[a + c*x^4]) + (B*e^2*x*Sqrt[a + c*x^4])/(c^(3/2)*(Sqrt[a] + Sqrt
[c]*x^2)) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*c^(3/2
)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(c^(7/4)*Sqrt[a + c*x^4]) + ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4]) + (a
^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[a + c*
x^4]) + (e*(2*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)
*c^(5/4)*Sqrt[a + c*x^4]) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2 - (Sqrt[c]*(A*c
*d^2 - 2*a*B*d*e - a*A*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4])
```

Rubi [A]   time = 0.531518, antiderivative size = 694, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1721, 1179, 1198, 220, 1196, 305}

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\big|\frac{1}{2}\right)\left(-\frac{\sqrt{c}\left(-aAe^2-2aBde+Acd^2\right)}{\sqrt{a}}-aBe^2+2Acde+Bcd^2\right)}{4a^{3/4}c^{7/4}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{}{(\sqrt{}}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*c*d^2 - 2*a*B*d*e - a*A*e^2 + (B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x^2))/(
2*a*c*Sqrt[a + c*x^4]) + (B*e^2*x*Sqrt[a + c*x^4])/(c^(3/2)*(Sqrt[a] + Sqrt
[c]*x^2)) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*c^(3/2
)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(c^(7/4)*Sqrt[a + c*x^4]) + ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4]) + (a
^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[a + c*
x^4]) + (e*(2*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)
*c^(5/4)*Sqrt[a + c*x^4]) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2 - (Sqrt[c]*(A*c
*d^2 - 2*a*B*d*e - a*A*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4])
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^2}{\left(a + cx^4\right)^{3/2}}\, dx = \int \left(\frac{Acd^2 - 2aBde - aAe^2 + \left(Bcd^2 + 2Acde - aBe^2\right)x^2}{c\left(a + cx^4\right)^{3/2}} + \frac{e(2Bd + Ae)}{c\sqrt{a + cx^4}} + \frac{Be^2x^2}{c\sqrt{a + cx^4}} \right) d$$

$$= \frac{\int \frac{Acd^2 - 2aBde - aAe^2 + \left(Bcd^2 + 2Acde - aBe^2\right)x^2}{\left(a + cx^4\right)^{3/2}}\, dx}{c} + \frac{\left(Be^2\right)\int \frac{x^2}{\sqrt{a + cx^4}}\, dx}{c} + \frac{(e(2Bd + Ae))\int \frac{1}{\sqrt{a + cx^4}}\, d}{c}$$

$$= \frac{x\left(Acd^2 - 2aBde - aAe^2 + \left(Bcd^2 + 2Acde - aBe^2\right)x^2\right)}{2ac\sqrt{a + cx^4}} + \frac{e(2Bd + Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{}}{2\sqrt[4]{ac^{5/4}}}$$

$$= \frac{x\left(Acd^2 - 2aBde - aAe^2 + \left(Bcd^2 + 2Acde - aBe^2\right)x^2\right)}{2ac\sqrt{a + cx^4}} + \frac{Be^2x\sqrt{a + cx^4}}{c^{3/2}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}Be^2\left(\sqrt{}\right)}{}$$

$$= \frac{x\left(Acd^2 - 2aBde - aAe^2 + \left(Bcd^2 + 2Acde - aBe^2\right)x^2\right)}{2ac\sqrt{a + cx^4}} + \frac{Be^2x\sqrt{a + cx^4}}{c^{3/2}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\left(Bcd^2 + 2}{2a}$$

Mathematica [C]   time = 0.178408, size = 166, normalized size = 0.24

$$\frac{2x^3\sqrt{\frac{cx^4}{a} + 1}\,_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{cx^4}{a}\right)\left(-3aBe^2 + 2Acde + Bcd^2\right) + 3x\sqrt{\frac{cx^4}{a} + 1}\,_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right)\left(aAe^2 + 2aBde + Acd^2\right) + 3}{6ac\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x]
```

```
[Out] (3*A*(c*d^2 - a*e^2)*x + 6*a*B*e*x*(-d + e*x^2) + 3*(A*c*d^2 + 2*a*B*d*e +
a*A*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a
)] + 2*(B*c*d^2 + 2*A*c*d*e - 3*a*B*e^2)*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeome
tric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.006, size = 458, normalized size = 0.7

$$Be^2\left(-\frac{x^3}{2c}\frac{1}{\sqrt{\left(x^4 + \frac{a}{c}\right)c}} + \frac{3i}{2}\sqrt{a}\sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x)
```

```
[Out] B*e^2*(-1/2/c*x^3/((x^4+a/c)*c)^(1/2)+3/2*I/c^(3/2)*a^(1/2)/(I/a^(1/2)*c^(1
/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/
(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^
(1/2)*c^(1/2))^(1/2),I)))+(A*e^2+2*B*d*e)*(-1/2/c*x/((x^4+a/c)*c)^(1/2)+1/2
/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
+(2*A*d*e+B*d^2)*(1/2/a*x^3/((x^4+a/c)*c)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
```

```
)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Ellipti
cE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+A*d^2*(1/2/a*x/((x^4+a/c)*c)^(1/2)+1/2/
a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^2}{\left(cx^4 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Be^2x^6 + \left(2\,Bde + Ae^2\right)x^4 + Ad^2 + \left(Bd^2 + 2\,Ade\right)x^2\right)\sqrt{cx^4 + a}}{c^2x^8 + 2\,acx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^6 + (2*B*d*e + A*e^2)*x^4 + A*d^2 + (B*d^2 + 2*A*d*e)*x^2
)*sqrt(c*x^4 + a)/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^2}{\left(a + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)*(d + e*x**2)**2/(a + c*x**4)**(3/2), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^2}{\left(cx^4 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + a)^(3/2), x)
```

3.10 $\displaystyle\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=395

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\sqrt{a}\sqrt{c}(Ae+Bd)-aBe+Acd\right)}{4a^{5/4}c^{5/4}\sqrt{a+cx^4}}+\frac{Be\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{2\sqrt[4]{ac}^{5/4}\sqrt{}}$$

```
[Out] (x*(A*c*d - a*B*e + c*(B*d + A*e)*x^2))/(2*a*c*Sqrt[a + c*x^4]) - ((B*d + A
*e)*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + ((B*d + A*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]
) + (B*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(5/4)*Sqrt[a
+ c*x^4]) + ((A*c*d - a*B*e - Sqrt[a]*Sqrt[c]*(B*d + A*e))*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rubi [A]   time = 0.327119, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1721, 1179, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\frac{1}{2}\right)\left(-\sqrt{a}\sqrt{c}(Ae+Bd)-aBe+Acd\right)}{4a^{5/4}c^{5/4}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Ae+Bd)E}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*c*d - a*B*e + c*(B*d + A*e)*x^2))/(2*a*c*Sqrt[a + c*x^4]) - ((B*d + A
*e)*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + ((B*d + A*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]
) + (B*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(5/4)*Sqrt[a
+ c*x^4]) + ((A*c*d - a*B*e - Sqrt[a]*Sqrt[c]*(B*d + A*e))*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
```

```
*p]
```

<u>Rule 1198</u>

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

<u>Rule 220</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

<u>Rule 1196</u>

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

<u>Rubi steps</u>

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)}{\left(a + cx^4\right)^{3/2}} dx = \int \left(\frac{Acd - aBe + c(Bd + Ae)x^2}{c\left(a + cx^4\right)^{3/2}} + \frac{Be}{c\sqrt{a + cx^4}} \right) dx$$

$$= \frac{\int \frac{Acd - aBe + c(Bd + Ae)x^2}{\left(a + cx^4\right)^{3/2}} dx}{c} + \frac{(Be) \int \frac{1}{\sqrt{a + cx^4}} dx}{c}$$

$$= \frac{x\left(Acd - aBe + c(Bd + Ae)x^2\right)}{2ac\sqrt{a + cx^4}} + \frac{Be\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}} F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{ac^{5/4}\sqrt{a + cx^4}}} - \int$$

$$= \frac{x\left(Acd - aBe + c(Bd + Ae)x^2\right)}{2ac\sqrt{a + cx^4}} + \frac{Be\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}} F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{ac^{5/4}\sqrt{a + cx^4}}} + \frac{(B}{}$$

$$= \frac{x\left(Acd - aBe + c(Bd + Ae)x^2\right)}{2ac\sqrt{a + cx^4}} - \frac{(Bd + Ae)x\sqrt{a + cx^4}}{2a\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} + \frac{(Bd + Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+}{\left(\sqrt{a}+}}}{2a^{3/4}c^{3/4}\sqrt{a}}$$

<u>Mathematica [C]</u>   time = 0.113368, size = 126, normalized size = 0.32

$$\frac{2cx^3\sqrt{\frac{cx^4}{a}+1}\,_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^4}{a}\right)(Ae + Bd) + 3x\sqrt{\frac{cx^4}{a}+1}\,_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)(aBe + Acd) + 3x(Acd - aBe)}{6ac\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x]
```

```
[Out] (3*(A*c*d - a*B*e)*x + 3*(A*c*d + a*B*e)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometr
ic2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*(B*d + A*e)*x^3*Sqrt[1 + (c*x^4)/a
```

```
]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.006, size = 320, normalized size = 0.8

$$Be\left(-\frac{x}{2c}\frac{1}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{1}{2c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\right)+(Ae+Bd)\left(\frac{x^3}{2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] B*e*(-1/2/c*x/((x^4+a/c)*c)^(1/2)+1/2/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+(A*e+B*d)*(1/2/a*x^3/((x^4+a/c)*c)^(1
/2)-1/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^
(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+A*d*(1/2
/a*x/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^2+A\right)\left(ex^2+d\right)}{\left(cx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bex^4+(Bd+Ae)x^2+Ad\right)\sqrt{cx^4+a}}{c^2x^8+2\,acx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + a)/(c^2*x^8 + 2*a*c
*x^4 + a^2), x)
```

Sympy [C]   time = 15.8711, size = 167, normalized size = 0.42

$$\frac{Adx\Gamma\left(\frac{1}{4}\right)_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right)_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right)_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right)_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] A*d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper((3/4,
3/2), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + B*e*x**5*
gamma(5/4)*hyper((5/4, 3/2), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*
gamma(9/4))
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)}{\left(cx^4 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

3.11 $\int \dfrac{A+Bx^2}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B-A\sqrt{c}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right|}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (x*(A + B*x^2))/(2*a*Sqrt[a + c*x^4]) - (B*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^
(3/4)*c^(3/4)*Sqrt[a + c*x^4]) - ((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A]   time = 0.0964995, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1179, 1198, 220, 1196}

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B-A\sqrt{c}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}+\frac{x\left(A}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A + B*x^2))/(2*a*Sqrt[a + c*x^4]) - (B*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^
(3/4)*c^(3/4)*Sqrt[a + c*x^4]) - ((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(a + cx^4\right)^{3/2}}\, dx = \frac{x\left(A + Bx^2\right)}{2a\sqrt{a + cx^4}} - \frac{\int \frac{-A+Bx^2}{\sqrt{a+cx^4}}\, dx}{2a}$$

$$= \frac{x\left(A + Bx^2\right)}{2a\sqrt{a + cx^4}} + \frac{\left(A - \frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\, dx}{2a} + \frac{B\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\, dx}{2\sqrt{a}\sqrt{c}}$$

$$= \frac{x\left(A + Bx^2\right)}{2a\sqrt{a + cx^4}} - \frac{Bx\sqrt{a + cx^4}}{2a\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} + \frac{B\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a + cx^4}} + \frac{\left(A - \frac{\sqrt{}}{\sqrt{}}\right)}{}$$

Mathematica [C]   time = 0.0417099, size = 99, normalized size = 0.38

$$\frac{3Ax\sqrt{\frac{cx^4}{a} + 1}\,_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{cx^4}{a}\right) + 2Bx^3\sqrt{\frac{cx^4}{a} + 1}\,_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{7}{4}; -\frac{cx^4}{a}\right) + 3Ax}{6a\sqrt{a + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + c*x^4)^(3/2),x]
```

```
[Out] (3*A*x + 3*A*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^
4)/a)] + 2*B*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*
x^4)/a)])/(6*a*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.004, size = 217, normalized size = 0.8

$$B\left(\frac{x^3}{2a}\frac{1}{\sqrt{\left(x^4 + \frac{a}{c}\right)c}} - \frac{i}{2}\sqrt{1 - ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1 + ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}, i\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+a)^(3/2),x)
```

```
[Out] B*(1/2/a*x^3/((x^4+a/c)*c)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/
2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)))+A*(1/2/a*x/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2
```

```
))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + a}\left(Bx^2 + A\right)}{c^2x^8 + 2\,acx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(B*x^2 + A)/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

Sympy [C]   time = 8.42081, size = 78, normalized size = 0.3

$$\frac{Ax\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+a)**(3/2),x)
```

```
[Out] A*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*exp_po
lar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + a)^(3/2), x)
```

3.12 $\int \dfrac{A+Bx^2}{\left(d+ex^2\right)\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=732

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\sqrt{a}\sqrt{c}(Bd-Ae)+aBe+Acd\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{2\sqrt[4]{a}\sqrt{a+cx^4}}$$

```
[Out] (x*(A*c*d + a*B*e + c*(B*d - A*e)*x^2))/(2*a*(c*d^2 + a*e^2)*Sqrt[a + c*x^4
]) - (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt
[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(3/2)) + (c^(1/4)
*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^2 + a*
e^2)*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a + c*x
^4]) + ((A*c*d + a*B*e - Sqrt[a]*Sqrt[c]*(B*d - A*e))*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (a
^(3/4)*e*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*
e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/
4)*d*(c^2*d^4 - a^2*e^4)*Sqrt[a + c*x^4])
```

Rubi [A]   time = 0.781061, antiderivative size = 732, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1721, 1179, 1198, 220, 1196, 1217, 1707}

$$\frac{a^{3/4}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{a+cx^4}\left(c^2d^4-a^2e^4\right)}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\,t\ldots\right.}{4a^{5/4}\sqrt[4]{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (x*(A*c*d + a*B*e + c*(B*d - A*e)*x^2))/(2*a*(c*d^2 + a*e^2)*Sqrt[a + c*x^4
]) - (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt
[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(3/2)) + (c^(1/4)
*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^2 + a*
e^2)*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a + c*x
^4]) + ((A*c*d + a*B*e - Sqrt[a]*Sqrt[c]*(B*d - A*e))*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (a
^(3/4)*e*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*
e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/
4)*d*(c^2*d^4 - a^2*e^4)*Sqrt[a + c*x^4])
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)])*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)\left(a + cx^4\right)^{3/2}}\, dx = \int \left(\frac{Acd + aBe + c(Bd - Ae)x^2}{\left(cd^2 + ae^2\right)\left(a + cx^4\right)^{3/2}} + \frac{e(-Bd + Ae)}{\left(cd^2 + ae^2\right)\left(d + ex^2\right)\sqrt{a + cx^4}}\right) dx$$

$$= \frac{\int \frac{Acd + aBe + c(Bd - Ae)x^2}{\left(a + cx^4\right)^{3/2}}\, dx}{cd^2 + ae^2} - \frac{(e(Bd - Ae))\int \frac{1}{(d + ex^2)\sqrt{a + cx^4}}\, dx}{cd^2 + ae^2}$$

$$= \frac{x\left(Acd + aBe + c(Bd - Ae)x^2\right)}{2a\left(cd^2 + ae^2\right)\sqrt{a + cx^4}} - \frac{\int \frac{-Acd - aBe + c(Bd - Ae)x^2}{\sqrt{a + cx^4}}\, dx}{2a\left(cd^2 + ae^2\right)} - \frac{\left(\sqrt{c}e(Bd - Ae)\right)\int \frac{1}{\sqrt{a + cx^4}}\, dx}{\left(\sqrt{c}d - \sqrt{ae}\right)\left(cd^2 + ae^2\right)} +$$

$$= \frac{x\left(Acd + aBe + c(Bd - Ae)x^2\right)}{2a\left(cd^2 + ae^2\right)\sqrt{a + cx^4}} - \frac{e^{3/2}(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2 + ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a + cx^4}}\right)}{2\sqrt{d}\left(cd^2 + ae^2\right)^{3/2}} - \frac{\sqrt[4]{c}e(Bd - Ae)\left(\sqrt{a} + \right.}{2\sqrt[4]{a}\left(\sqrt{c}\right.}$$

$$= \frac{x\left(Acd + aBe + c(Bd - Ae)x^2\right)}{2a\left(cd^2 + ae^2\right)\sqrt{a + cx^4}} - \frac{\sqrt{c}(Bd - Ae)x\sqrt{a + cx^4}}{2a\left(cd^2 + ae^2\right)\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e^{3/2}(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2}}{\sqrt{d}\sqrt{e}}\right.}{2\sqrt{d}\left(cd^2 + ae^2\right)^{3/2}}$$

Mathematica [C]   time = 0.80458, size = 432, normalized size = 0.59

$$d\sqrt{\frac{cx^4}{a} + 1}\left(\sqrt{a}B - iA\sqrt{c}\right)\left(\sqrt{c}d - i\sqrt{ae}\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right), -1\right) - \sqrt{a}\sqrt{c}d\sqrt{\frac{cx^4}{a} + 1}(Bd - Ae)E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^2*x + a*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e*x
+ B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^2*x^3 - A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e
*x^3 - Sqrt[a]*Sqrt[c]*d*(B*d - A*e)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (Sqrt[a]*B - I*A*Sqrt[c])*d*(Sqrt[c]*
d - I*Sqrt[a]*e)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/S
qrt[a]]*x], -1] + (2*I)*a*B*d*e*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a
]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*I)*a*A*e
^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*(c*d^2 +
 a*e^2)*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.023, size = 564, normalized size = 0.8

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x)
```

```
[Out] B/e*(1/2*a*x/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+(A*e-B*d)/e*(-2*c*(1/4/a*e/(a*e^2+c*d^
```

```
2)*x^3-1/4/a*d/(a*e^2+c*d^2)*x)/((x^4+a/c)*c)^(1/2)+1/2/a*c*d/(a*e^2+c*d^2)
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/
2*I/a^(1/2)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I/a^(1/2)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/(a*e^
2+c*d^2)*e^2/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2)))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)), x)

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+a)**(3/2),x)

[Out] Timed out

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

3.13 $\int \dfrac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=1494

<div align="center">result too large to display</div>

```
[Out] (c*x*(A*c*d^2 + 2*a*B*d*e - a*A*e^2 + (B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x^2))
/(2*a*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (Sqrt[c]*e^2*(B*d - A*e)*x*Sqrt[
a + c*x^4])/(2*d*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (Sqrt[c]*(B*c
*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)^2*(Sqrt
[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e
^2)^2*(d + e*x^2)) - (e^(3/2)*(B*d - A*e)*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*
d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(4*d^(3/2)*(c*d^2 + a*e
^2)^(5/2)) - (e^(3/2)*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*ArcTan[(Sqrt[c*d^2 +
a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(5
/2)) - (a^(1/4)*c^(1/4)*e^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*d*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/4)*(B*c*d^2 - 2*A*c*d*
e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^2 + a*
e^2)^2*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a +
 c*x^4]) - (c^(1/4)*e*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqr
t[a + c*x^4]) - (c^(1/4)*(B*c*d^2 - 2*A*c*d*e - a*B*e^2 - (Sqrt[c]*(A*c*d^2
 + 2*a*B*d*e - a*A*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(3/4)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(
B*d - A*e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt
[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt
[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (a^(3/4)*e*((Sqrt[c
]*d)/Sqrt[a] + e)^2*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[
a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^
(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

<u>Rubi [A]</u>   time = 2.0065, antiderivative size = 1494, normalized size of antiderivative =
1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$
= 0.357, Rules used = {1721, 1179, 1198, 220, 1196, 1224, 1715, 1709, 1707, 1217}

<div align="center">result too large to display</div>

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*(a + c*x^4)^(3/2)),x]
```

```
[Out] (c*x*(A*c*d^2 + 2*a*B*d*e - a*A*e^2 + (B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x^2))
/(2*a*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (Sqrt[c]*e^2*(B*d - A*e)*x*Sqrt[
a + c*x^4])/(2*d*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (Sqrt[c]*(B*c
*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)^2*(Sqrt
[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e
^2)^2*(d + e*x^2)) - (e^(3/2)*(B*d - A*e)*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*
```

d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(4*d^(3/2)*(c*d^2 + a*e
^2)^(5/2)) - (e^(3/2)*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*ArcTan[(Sqrt[c*d^2 +
a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(5
/2)) - (a^(1/4)*c^(1/4)*e^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*d*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/4)*(B*c*d^2 - 2*A*c*d*
e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^2 + a*
e^2)^2*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a +
 c*x^4]) - (c^(1/4)*e*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqr
t[a + c*x^4]) - (c^(1/4)*(B*c*d^2 - 2*A*c*d*e - a*B*e^2 - (Sqrt[c]*(A*c*d^2
 + 2*a*B*d*e - a*A*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(3/4)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(
B*d - A*e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt
[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt
[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (a^(3/4)*e*((Sqrt[c
]*d)/Sqrt[a] + e)^2*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[
a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^
(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])

Rule 1721

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(

```
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^2 \left(a + cx^4\right)^{3/2}} \, dx = \int \left(\frac{c\left(Acd^2 + 2aBde - aAe^2 + \left(Bcd^2 - 2Acde - aBe^2\right)x^2\right)}{\left(cd^2 + ae^2\right)^2 \left(a + cx^4\right)^{3/2}} + \frac{e(-Bd + Ae)}{\left(cd^2 + ae^2\right)\left(d + ex^2\right)^2 \sqrt{a + }} \right.$$

$$= \frac{c \int \frac{Acd^2 + 2aBde - aAe^2 + \left(Bcd^2 - 2Acde - aBe^2\right)x^2}{\left(a + cx^4\right)^{3/2}} \, dx}{\left(cd^2 + ae^2\right)^2} - \frac{(e(Bd - Ae)) \int \frac{1}{\left(d + ex^2\right)^2 \sqrt{a + cx^4}} \, dx}{cd^2 + ae^2} - \frac{\left(e\left(Bcd^2 - 2\right.\right.}{}$$

$$= \frac{cx\left(Acd^2 + 2aBde - aAe^2 + \left(Bcd^2 - 2Acde - aBe^2\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^2 \sqrt{a + cx^4}} - \frac{e^3(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)^2 \left(d + ex^2\right)} - \frac{c \int}{}$$

$$= \frac{cx\left(Acd^2 + 2aBde - aAe^2 + \left(Bcd^2 - 2Acde - aBe^2\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^2 \sqrt{a + cx^4}} - \frac{e^3(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)^2 \left(d + ex^2\right)} - \frac{e^{3/2}}{}$$

$$= \frac{cx\left(Acd^2 + 2aBde - aAe^2 + \left(Bcd^2 - 2Acde - aBe^2\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^2 \sqrt{a + cx^4}} + \frac{\sqrt{c}e^2(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)^2 \left(\sqrt{a} + \sqrt{c}x^2\right)} -$$

$$= \frac{cx\left(Acd^2 + 2aBde - aAe^2 + \left(Bcd^2 - 2Acde - aBe^2\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^2 \sqrt{a + cx^4}} + \frac{\sqrt{c}e^2(Bd - Ae)x\sqrt{a + cx^4}}{2d\left(cd^2 + ae^2\right)^2 \left(\sqrt{a} + \sqrt{c}x^2\right)} -$$

Mathematica [C]   time = 1.62582, size = 427, normalized size = 0.29

$$d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(cdx\left(d + ex^2\right)\left(-aAe^2 + aBe\left(2d - ex^2\right) + Acd\left(d - 2ex^2\right) + Bcd^2x^2\right) + ae^3x\left(a + cx^4\right)(Ae - Bd)\right) - \sqrt{\frac{cx^4}{a} + 1}\left(d + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*(a + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*(a*e^3*(-(B*d) + A*e)*x*(a + c*x^4) + c*d*x*(d
 + e*x^2)*(-(a*A*e^2) + B*c*d^2*x^2 + A*c*d*(d - 2*e*x^2) + a*B*e*(2*d - e*
x^2))) - (d + e*x^2)*Sqrt[1 + (c*x^4)/a]*(-(Sqrt[a]*Sqrt[c]*d*(-(B*c*d^3) +
 2*A*c*d^2*e + 2*a*B*d*e^2 - a*A*e^3)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/
Sqrt[a]]*x], -1]) + I*(Sqrt[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(A*c*d^2 + I*Sqr
t[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(2*B*d - A*e))*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] + a*e*(-5*B*c*d^3 + 7*A*c*d^2*e + a*B*d*e^2 + a*
A*e^3)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/
Sqrt[a]]*x], -1])))/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^3 + a*d*e^2)^2*(d +
 e*x^2)*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.033, size = 1384, normalized size = 0.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x)
```

```
[Out] (A*e-B*d)/e*(1/2*e^4/(a*e^2+c*d^2)^2/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-2*c*(1/2
/a*c*d*e/(a*e^2+c*d^2)^2*x^3+1/4/a*(a*e^2-c*d^2)/(a*e^2+c*d^2)^2*x)/((x^4+a
/c)*c)^(1/2)-1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2
))^(1/2),I)*e^2*c/(a*e^2+c*d^2)^2+1/2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)/a*c^2/(a*e^2+c*d^2)^2*d^2-1/2*I*a^(1/2)/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d*EllipticF(x*(I
/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(
1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(
1/2)*e^3/(a*e^2+c*d^2)^2/d*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I/a^(1/
2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*d*e/(a*e^2+c*d^2)^2*EllipticF(x*(
I/a^(1/2)*c^(1/2))^(1/2),I)-I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2
)*d*e/(a*e^2+c*d^2)^2*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*e^4/(a*e
^2+c*d^2)^2/d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1
/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1
/2))^(1/2))*a+7/2*e^2/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))
^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c)+B/e*(-2*c*(1/4/a*e/(a*e^2+c*d^2)*x^3-1
/4/a*d/(a*e^2+c*d^2)*x)/((x^4+a/c)*c)^(1/2)+1/2/a*c*d/(a*e^2+c*d^2)/(I/a^(1
/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2
)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I/a^(1
/2)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)-1/2*I/a^(1/2)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a^(1/2)*c
^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/(a*e^2+c*d^2)
*e^2/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2
),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2
)))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

3.14 $\int \dfrac{A+Bx^2}{\left(d+ex^2\right)^3\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=2452

<div align="center">result too large to display</div>

[Out] (c*x*(A*c*d*(c*d^2 - 3*a*e^2) + a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 - 3*A*
c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x^2))/(2*a*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^
4]) + (3*Sqrt[c]*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^
2*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^2*(B*c*d^2 - 2*A*
c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[
c]*x^2)) - (c^(3/2)*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x*Sqrt[
a + c*x^4])/(2*a*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A
*e)*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)^2*(d + e*x^2)^2) - (3*e^3*(B*d
- A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^3*(d + e
x^2)) - (e^3(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^
2 + a*e^2)^3*(d + e*x^2)) - (e^(3/2)*(3*c*d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e
 - a*B*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4]
)]])/(4*d^(3/2)*(c*d^2 + a*e^2)^(7/2)) - (c*e^(3/2)*(B*c*d^3 - 3*A*c*d^2*e -
 3*a*B*d*e^2 + a*A*e^3)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqr
t[a + c*x^4]]])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(7/2)) - (3*e^(3/2)*(B*d - A*e)*
(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[
d]*Sqrt[e]*Sqrt[a + c*x^4])]])/(16*d^(5/2)*(c*d^2 + a*e^2)^(7/2)) - (3*a^(1/
4)*c^(1/4)*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*e^2*
(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*
d*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (c^(5/4)*(B*c*d^3 - 3*A*c*d^2*e - 3*
a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*
d^2 + a*e^2)^3*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(4*c*d^2 - Sqrt[a]
*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*
d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) - (c^(1/4)*e
*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2
*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) - (c^
(5/4)*e*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^3*S
qrt[a + c*x^4]) + (c^(3/4)*(A*c^2*d^3 - a^2*B*e^3 - Sqrt[a]*c^(3/2)*d^2*(B*
d - 3*A*e) + 3*a*c*d*e*(B*d - A*e) + a^(3/2)*Sqrt[c]*e^2*(3*B*d - A*e))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d^2 + a*e^2)^3*Sqrt[a +
c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e
 - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e
)*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (a^(3/4)*c^(3/4)*e*((Sqrt[c]*d)/Sqrt
[a] + e)^2*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d
- Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(4*d*(c*d^2 - a*e^2)*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (3*e*(Sqrt[c]*d
 + Sqrt[a]*e)*(B*d - A*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[
c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^3*Sq

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rt[a + c*x^4])

Rubi [A]   time = 3.97126, antiderivative size = 2452, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1721, 1179, 1198, 220, 1196, 1224, 1697, 1715, 1709, 1707, 1217}

result too large to display

Antiderivative was successfully verified.

[In] Int[(A + B*x^2)/((d + e*x^2)^3*(a + c*x^4)^(3/2)),x]

[Out] (c*x*(A*c*d*(c*d^2 - 3*a*e^2) + a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 - 3*A*
c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x^2))/(2*a*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^
4]) + (3*Sqrt[c]*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^
2*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^2*(B*c*d^2 - 2*A*
c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[
c]*x^2)) - (c^(3/2)*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x*Sqrt[
a + c*x^4])/(2*a*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A
*e)*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)^2*(d + e*x^2)^2) - (3*e^3*(B*d
- A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^3*(d + e
x^2)) - (e^3(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^
2 + a*e^2)^3*(d + e*x^2)) - (e^(3/2)*(3*c*d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e
 - a*B*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4]
)])/(4*d^(3/2)*(c*d^2 + a*e^2)^(7/2)) - (c*e^(3/2)*(B*c*d^3 - 3*A*c*d^2*e -
 3*a*B*d*e^2 + a*A*e^3)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqr
t[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(7/2)) - (3*e^(3/2)*(B*d - A*e)*
(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[
d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2)*(c*d^2 + a*e^2)^(7/2)) - (3*a^(1/
4)*c^(1/4)*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*e^2*
(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*
d*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (c^(5/4)*(B*c*d^3 - 3*A*c*d^2*e - 3*
a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*
d^2 + a*e^2)^3*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(4*c*d^2 - Sqrt[a]
*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*
d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) - (c^(1/4)*e
*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2
*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) - (c^
(5/4)*e*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^3*S
qrt[a + c*x^4]) + (c^(3/4)*(A*c^2*d^3 - a^2*B*e^3 - Sqrt[a]*c^(3/2)*d^2*(B*
d - 3*A*e) + 3*a*c*d*e*(B*d - A*e) + a^(3/2)*Sqrt[c]*e^2*(3*B*d - A*e))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d^2 + a*e^2)^3*Sqrt[a +
c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e
 - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e
)*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (a^(3/4)*c^(3/4)*e*((Sqrt[c]*d)/Sqrt
[a] + e)^2*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[

c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d
 - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(4*d*(c*d^2 - a*e^2)*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (3*e*(Sqrt[c]*d
 + Sqrt[a]*e)*(B*d - A*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[
c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^3*Sq
rt[a + c*x^4])

Rule 1721

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]

Rule 1179

Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]

Rule 1198

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]

Rule 220

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

Rule 1196

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)])*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]

Rule 1224

Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]

Rule 1697

Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
d(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int

```
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^3 \left(a + cx^4\right)^{3/2}}\, dx = \int \left(\frac{c\left(Acd\left(cd^2 - 3ae^2\right) + aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 - 3Acd^2e - 3aBde^2 + aAe^3\right)x^2\right)}{\left(cd^2 + ae^2\right)^3 \left(a + cx^4\right)^{3/2}} \right)$$

$$= \frac{c\int \frac{Acd\left(cd^2 - 3ae^2\right) + aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 - 3Acd^2e - 3aBde^2 + aAe^3\right)x^2}{\left(a + cx^4\right)^{3/2}}\, dx}{\left(cd^2 + ae^2\right)^3} - \frac{\left(e(Bd - Ae)\right)\int \frac{1}{\left(d + ex^2\right)^3 \sqrt{a+}}}{cd^2 + ae^2}$$

$$= \frac{cx\left(Acd\left(cd^2 - 3ae^2\right) + aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 - 3Acd^2e - 3aBde^2 + aAe^3\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^3 \sqrt{a + cx^4}} -$$

$$= \frac{cx\left(Acd\left(cd^2 - 3ae^2\right) + aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 - 3Acd^2e - 3aBde^2 + aAe^3\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^3 \sqrt{a + cx^4}} -$$

$$= \frac{cx\left(Acd\left(cd^2 - 3ae^2\right) + aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 - 3Acd^2e - 3aBde^2 + aAe^3\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^3 \sqrt{a + cx^4}} +$$

$$= \frac{cx\left(Acd\left(cd^2 - 3ae^2\right) + aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 - 3Acd^2e - 3aBde^2 + aAe^3\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^3 \sqrt{a + cx^4}} +$$

$$= \frac{cx\left(Acd\left(cd^2 - 3ae^2\right) + aBe\left(3cd^2 - ae^2\right) + c\left(Bcd^3 - 3Acd^2e - 3aBde^2 + aAe^3\right)x^2\right)}{2a\left(cd^2 + ae^2\right)^3 \sqrt{a + cx^4}} +$$

Mathematica [C]   time = 3.03304, size = 630, normalized size = 0.26

$$dx \sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(4cd^2\left(d + ex^2\right)^2\left(B\left(-a^2e^3 + 3acde\left(d - ex^2\right) + c^2d^3x^2\right) + Ac\left(ae^2\left(ex^2 - 3d\right) + cd^2\left(d - 3ex^2\right)\right)\right) - 2ade^3\left(a + c\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^3*(a + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*x*(-2*a*d*e^3*(B*d - A*e)*(c*d^2 + a*e^2)*(a +
 c*x^4) + a*e^3*(-13*B*c*d^3 + 17*A*c*d^2*e + a*B*d*e^2 + 3*a*A*e^3)*(d + e
*x^2)*(a + c*x^4) + 4*c*d^2*(d + e*x^2)^2*(B*(-(a^2*e^3) + c^2*d^3*x^2 + 3*
a*c*d*e*(d - e*x^2)) + A*c*(c*d^2*(d - 3*e*x^2) + a*e^2*(-3*d + e*x^2)))) -
 (d + e*x^2)^2*Sqrt[1 + (c*x^4)/a]*(Sqrt[a]*Sqrt[c]*d*(3*A*e*(-4*c^2*d^4 +
7*a*c*d^2*e^2 + a^2*e^4) + B*(4*c^2*d^5 - 25*a*c*d^3*e^2 + a^2*d*e^4))*Elli
pticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(Sqrt[c]*d
 - I*Sqrt[a]*e)*(4*A*c^2*d^4 + (4*I)*Sqrt[a]*c^(3/2)*d^3*(B*d - 2*A*e) + 19
*a*c*d^2*e*(B*d - A*e) - (2*I)*a^(3/2)*Sqrt[c]*d*e^2*(3*B*d - A*e) - a^2*e^
3*(B*d + 3*A*e))*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + a*
e*(3*A*e*(21*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + B*(-35*c^2*d^5 + 26*a*c*d
^3*e^2 + a^2*d*e^4))*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqr
t[(I*Sqrt[c])/Sqrt[a]]*x], -1])))/(8*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^3 + a
*d*e^2)^3*(d + e*x^2)^2*Sqrt[a + c*x^4])
```

Maple [C]   time = 0.035, size = 2326, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x)
```

```
[Out] B/e*(1/2*e^4/(a*e^2+c*d^2)^2/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-2*c*(1/2/a*c*d*e
/(a*e^2+c*d^2)^2*x^3+1/4/a*(a*e^2-c*d^2)/(a*e^2+c*d^2)^2*x)/((x^4+a/c)*c)^(
1/2)-1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I)*e^2*c/(a*e^2+c*d^2)^2+1/2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*c^(1/2))^(1/2),I)/a*c^2/(a*e^2+c*d^2)^2*d^2-1/2*I*a^(1/2)/(I/a^(1/
2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)
^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d*EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)+1/2*I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^3
/(a*e^2+c*d^2)^2/d*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I/a^(1/2)/(I/a^
(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*d*e/(a*e^2+c*d^2)^2*EllipticF(x*(I/a^(1/2
)*c^(1/2))^(1/2),I)-I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*d*e/(a
*e^2+c*d^2)^2*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*e^4/(a*e^2+c*d^2
)^2/d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/
2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/
2))*a+7/2*e^2/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(
I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(
I/a^(1/2)*c^(1/2))^(1/2))*c)+(A*e-B*d)/e*(1/4*e^4/(a*e^2+c*d^2)^2/d*x*(c*x^
4+a)^(1/2)/(e*x^2+d)^2+1/8*e^4*(3*a*e^2+17*c*d^2)/(a*e^2+c*d^2)^3/d^2*x*(c*
x^4+a)^(1/2)/(e*x^2+d)-2*c*(-1/4*c*e*(a*e^2-3*c*d^2)/a/(a*e^2+c*d^2)^3*x^3+
1/4*c*d*(3*a*e^2-c*d^2)/a/(a*e^2+c*d^2)^3*x)/((x^4+a/c)*c)^(1/2)-1/8/(I/a^(
1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^
2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*c*e^4/d/(
a*e^2+c*d^2)^3*a-27/8/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)*c^2*e^2*d/(a*e^2+c*d^2)^3+1/2/(I/a^(1/2)*c^(1/2))^(1/2)*
(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(
1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*c^3*d^3/a/(a*e^2+c*d^2)^3-21/
8*I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*e^3/(a*e^2+c*d^2)^3*Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2
)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)
^(1/2)*c^(5/2)*e/(a*e^2+c*d^2)^3*d^2*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),
I)-3/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(5/2)*e/(a*e^2+c*d^2)^3*d
^2*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*a^(3/2)/(I/a^(1/2)*c^(1/2
))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c
*x^4+a)^(1/2)*c^(1/2)*e^5/(a*e^2+c*d^2)^3/d^2*EllipticF(x*(I/a^(1/2)*c^(1/2
))^(1/2),I)+3/8*I*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^5/(a*e^2
+c*d^2)^3/d^2*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+21/8*I*a^(1/2)/(I/a^
(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*e^3/(a*e^2+c*d^2)^3*EllipticE(x*(I/a^(1/2
)*c^(1/2))^(1/2),I)+3/8*e^6/(a*e^2+c*d^2)^3/d^3/(I/a^(1/2)*c^(1/2))^(1/2)*(
```

```
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2
)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4*e^4/(a*e^2+c*d^2)^3/d/(
I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1
/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a*c+6
3/8*e^2/(a*e^2+c*d^2)^3*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^
(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^
(1/2)*c^(1/2))^(1/2))*c^2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

3.15 $\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx$

Optimal. Leaf size=169

$$\frac{x\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}+\frac{x\left(\frac{\sqrt{-a}B}{\sqrt{c}}+A\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};\right.}{2a}$$

```
[Out] ((A - (Sqrt[-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, -((Sq
rt[c]*x^2)/Sqrt[-a]), -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q) + ((A + (Sqrt[
-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (Sqrt[c]*x^2)/Sqr
t[-a], -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q)
```

Rubi [A]   time = 0.218909, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1693, 430, 429}

$$\frac{x\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}+\frac{x\left(\frac{\sqrt{-a}B}{\sqrt{c}}+A\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x]
```

```
[Out] ((A - (Sqrt[-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, -((Sq
rt[c]*x^2)/Sqrt[-a]), -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q) + ((A + (Sqrt[
-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (Sqrt[c]*x^2)/Sqr
t[-a], -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q)
```

Rule 1693

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^q}{a + cx^4}\, dx = \int \left(-\frac{\left(\sqrt{-a}B + A\sqrt{c}\right)\left(d + ex^2\right)^q}{2\sqrt{-a}\sqrt{c}\left(\sqrt{-a} - \sqrt{c}x^2\right)} + \frac{\left(\sqrt{-a}B - A\sqrt{c}\right)\left(d + ex^2\right)^q}{2\sqrt{-a}\sqrt{c}\left(\sqrt{-a} + \sqrt{c}x^2\right)}\right) dx$$

$$= -\left(\frac{1}{2}\left(\frac{A}{\sqrt{-a}} + \frac{B}{\sqrt{c}}\right)\int \frac{\left(d + ex^2\right)^q}{\sqrt{-a} - \sqrt{c}x^2}\, dx\right) + \frac{1}{2}\left(\frac{aA}{(-a)^{3/2}} + \frac{B}{\sqrt{c}}\right)\int \frac{\left(d + ex^2\right)^q}{\sqrt{-a} + \sqrt{c}x^2}\, dx$$

$$= -\left(\frac{1}{2}\left(\left(\frac{A}{\sqrt{-a}} + \frac{B}{\sqrt{c}}\right)\left(d + ex^2\right)^q\left(1 + \frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1 + \frac{ex^2}{d}\right)^q}{\sqrt{-a} - \sqrt{c}x^2}\, dx\right) + \frac{1}{2}\left(\left(\frac{aA}{(-a)^{3/2}} + \frac{B}{\sqrt{c}}\right)(d + $$

$$= \frac{\left(A - \frac{\sqrt{-a}B}{\sqrt{c}}\right)x\left(d + ex^2\right)^q\left(1 + \frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2}; 1, -q; \frac{3}{2}; -\frac{\sqrt{c}x^2}{\sqrt{-a}}, -\frac{ex^2}{d}\right)}{2a} - \frac{\left(\frac{A}{\sqrt{-a}} + \frac{B}{\sqrt{c}}\right)x\left(d + ex^2\right)^q}{}$$

Mathematica [F]   time = 0.395585, size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^q}{a + cx^4}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x]
```

```
[Out] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4), x]
```

Maple [F]   time = 0.066, size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x)
```

```
[Out] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

3.16 $\displaystyle\int \frac{2+x^2}{\left(1+x^2\right)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{2}\left(x^2+2\right)E\left(\tan^{-1}(x)\Big|\frac{1}{2}\right)}{\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (Sqrt[2]*(2 + x^2)*EllipticE[ArcTan[x], 1/2])/(Sqrt[(2 + x^2)/(1 + x^2)]*Sq
rt[2 + 3*x^2 + x^4])
```

Rubi [A]   time = 0.0290589, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1456, 411}

$$\frac{\sqrt{2}\left(x^2+2\right)E\left(\tan^{-1}(x)\Big|\frac{1}{2}\right)}{\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((1 + x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[2]*(2 + x^2)*EllipticE[ArcTan[x], 1/2])/(Sqrt[(2 + x^2)/(1 + x^2)]*Sq
rt[2 + 3*x^2 + x^4])
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{2+x^2}{\left(1+x^2\right)\sqrt{2+3x^2+x^4}}\,dx = \frac{\left(\sqrt{1+x^2}\sqrt{2+x^2}\right)\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{2+3x^2+x^4}}$$

$$= \frac{\sqrt{2}\left(2+x^2\right)E\left(\tan^{-1}(x)\Big|\frac{1}{2}\right)}{\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C]   time = 0.135588, size = 94, normalized size = 1.96

$$\frac{-i\sqrt{x^2+1}\sqrt{x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+x^3+i\sqrt{x^2+1}\sqrt{x^2+2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2x}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((1 + x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (2*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt
[2 + 3*x^2 + x^4]
```

Maple [C]   time = 0.028, size = 81, normalized size = 1.7

$$\left(x^2+2\right)x\frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}-\frac{i}{2}\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] (x^2+2)*x/((x^2+1)*(x^2+2))^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/
2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*
x*2^(1/2),2^(1/2)))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+2}{\sqrt{x^4+3x^2+2}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^4 + 3*x^2 + 2)*(x^2 + 1)), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{x^4+2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 2}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(x**2+1)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((x**2 + 2)/(sqrt((x**2 + 1)*(x**2 + 2))*(x**2 + 1)), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 + 2}{\sqrt{x^4 + 3\,x^2 + 2}\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^4 + 3*x^2 + 2)*(x^2 + 1)), x)
```

3.17 $\quad\int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=755

$$\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}\left(7Ac\left(4abe^3-15acde^2+15c^2d^3\right)-aBe\left(-ce(25ae+84bd)+24b^2\right.\right.}{\sqrt{a}}\right.$$

$$210c^{15/}$$

```
[Out] (e*(7*A*c*e*(15*c*d - 4*b*e) + B*(105*c^2*d^2 + 24*b^2*e^2 - c*e*(84*b*d +
25*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^3) + (e^2*(21*B*c*d - 6*b*B*e +
 7*A*c*e)*x^3*Sqrt[a + b*x^2 + c*x^4])/(35*c^2) + (B*e^3*x^5*Sqrt[a + b*x^2
 + c*x^4])/(7*c) + ((7*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*
e)) + B*(105*c^3*d^3 - 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e^2
*(21*b*d + 13*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(7/2)*(Sqrt[a] + Sqr
t[c]*x^2)) - (a^(1/4)*(7*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*
a*e)) + B*(105*c^3*d^3 - 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e
^2*(21*b*d + 13*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(105*c^(15/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(7*A*
c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)) + B*(105*c^3*d^3 - 48
*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e^2*(21*b*d + 13*a*e)) + (Sq
rt[c]*(7*A*c*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^3) - a*B*e*(105*c^2*d^2 +
 24*b^2*e^2 - c*e*(84*b*d + 25*a*e)))))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(15/4)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rubi [A]   time = 1.35688, antiderivative size = 755, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1679, 1197, 1103, 1195}

$$\frac{ex\sqrt{a+bx^2+cx^4}\left(B\left(-ce(25ae+84bd)+24b^2e^2+105c^2d^2\right)+7Ace(15cd-4be)\right)}{105c^3}+\frac{x\sqrt{a+bx^2+cx^4}\left(7Ace\left(-3ce(3a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*(7*A*c*e*(15*c*d - 4*b*e) + B*(105*c^2*d^2 + 24*b^2*e^2 - c*e*(84*b*d +
25*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^3) + (e^2*(21*B*c*d - 6*b*B*e +
 7*A*c*e)*x^3*Sqrt[a + b*x^2 + c*x^4])/(35*c^2) + (B*e^3*x^5*Sqrt[a + b*x^2
 + c*x^4])/(7*c) + ((7*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*
e)) + B*(105*c^3*d^3 - 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e^2
*(21*b*d + 13*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(7/2)*(Sqrt[a] + Sqr
t[c]*x^2)) - (a^(1/4)*(7*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*
a*e)) + B*(105*c^3*d^3 - 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e
^2*(21*b*d + 13*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(105*c^(15/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(7*A*
c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)) + B*(105*c^3*d^3 - 48
*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e^2*(21*b*d + 13*a*e)) + (Sq
rt[c]*(7*A*c*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^3) - a*B*e*(105*c^2*d^2 +
 24*b^2*e^2 - c*e*(84*b*d + 25*a*e))))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
```

```
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(15/4)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^3}{\sqrt{a + bx^2 + cx^4}}\, dx = \frac{Be^3x^5\sqrt{a + bx^2 + cx^4}}{7c} + \frac{\int \frac{7Acd^3 + 7cd^2(Bd + 3Ae)x^2 + e\left(21Bcd^2 + 21Acde - 5aBe^2\right)x^4 + e^2(21Bcd - 6bBe + 7Ace)x^6}{\sqrt{a + bx^2 + cx^4}}\, d}{7c}$$

$$= \frac{e^2(21Bcd - 6bBe + 7Ace)x^3\sqrt{a + bx^2 + cx^4}}{35c^2} + \frac{Be^3x^5\sqrt{a + bx^2 + cx^4}}{7c} + \frac{\int \frac{35Ac^2d^3 + \left(21Ace(5cd^2}{}}{}$$

$$= \frac{e\left(7Ace(15cd - 4be) + B\left(105c^2d^2 + 24b^2e^2 - ce(84bd + 25ae)\right)\right)x\sqrt{a + bx^2 + cx^4}}{105c^3} + \frac{e^2(21B}{}$$

$$= \frac{e\left(7Ace(15cd - 4be) + B\left(105c^2d^2 + 24b^2e^2 - ce(84bd + 25ae)\right)\right)x\sqrt{a + bx^2 + cx^4}}{105c^3} + \frac{e^2(21B}{}$$

$$= \frac{e\left(7Ace(15cd - 4be) + B\left(105c^2d^2 + 24b^2e^2 - ce(84bd + 25ae)\right)\right)x\sqrt{a + bx^2 + cx^4}}{105c^3} + \frac{e^2(21B}{}$$

```
2*I)*Sqrt[2]*b^3*B*(-b + Sqrt[b^2 - 4*a*c])*e^3*Sqrt[1 - (2*c*x^2)/(-b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I
*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4
*a*c])/(-b + Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b
 - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c
])])])/(c*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) - (((
63*I)/2)*a*A*c*(-b + Sqrt[b^2 - 4*a*c])*e^3*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b
^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*Arc
Sinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c
])/(-b + Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - S
qrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])
)/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) - (
(105*I)*A*c^3*d^3*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b -
 Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])
])/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) +
((105*I)*a*B*c^2*d^2*e*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
- (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/
(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*
a*c])])/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4
]) - ((42*I)*Sqrt[2]*a*b*B*c*d*e^2*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*
c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[
2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + S
qrt[b^2 - 4*a*c])])/(Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c
*x^4]) + ((105*I)*a*A*c^2*d*e^2*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt
[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^
2 + c*x^4]) + ((12*I)*Sqrt[2]*a*b^2*B*e^3*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2
 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSin
h[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/
(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*
x^2 + c*x^4]) - ((14*I)*Sqrt[2]*a*A*b*c*e^3*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b
^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c]
)/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a +
b*x^2 + c*x^4]) - ((25*I)*a^2*B*c*e^3*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sq
rt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b
+ Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a
+ b*x^2 + c*x^4]))/(105*c^3)
```

Maple [B]   time = 0.021, size = 1708, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] B*e^3*(1/7/c*x^5*(c*x^4+b*x^2+a)^(1/2)-6/35*b/c^2*x^3*(c*x^4+b*x^2+a)^(1/2)
+1/3*(-5/7*a/c+24/35*b^2/c^2)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(-5/7*a/c+24/3
5*b^2/c^2)/c*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^
(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(18/35*b/c^2*a-2/3*(-5/7*a/c+24/
35*b^2/c^2)/c*b)*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b
```

```
^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b
*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2
)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elliptic
E(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))))+(A*e^3+3*B*d*e^2)*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4
/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/15*b/c^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c
+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*
(-3/5*a/c+8/15*b^2/c^2)*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-
4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*
a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-E
llipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))))+(3*A*d*e^2+3*B*d^2*e)*(1/3/c*x*(c*x^4+b*x^2+a
)^(1/2)-1/12*a/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2
)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x
^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellip
ticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))-1/2*(3*A*d^2*e+B*d^3
)*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)
*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2)))+1/4*A*d^3*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2
)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x
^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Be^3x^8 + \left(3\,Bde^2 + Ae^3\right)x^6 + 3\left(Bd^2e + Ade^2\right)x^4 + Ad^3 + \left(Bd^3 + 3\,Ad^2e\right)x^2}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((B*e^3*x^8 + (3*B*d*e^2 + A*e^3)*x^6 + 3*(B*d^2*e + A*d*e^2)*x^4 +
A*d^3 + (B*d^3 + 3*A*d^2*e)*x^2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^3}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)*(d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

3.18 $\quad \int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=528

$$\dfrac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\dfrac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\dfrac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\tfrac{1}{4}\left(2-\dfrac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(-\dfrac{\sqrt{c}\left(2aBe(5cd-2be)-5Ac\left(3cd^2-ae^2\right)\right)}{\sqrt{a}}+B\left(-ce(9ae+2\right.\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (e*(10*B*c*d - 4*b*B*e + 5*A*c*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*
e^2*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + ((10*A*c*e*(3*c*d - b*e) + B*(15*c
^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(15*
c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(10*A*c*e*(3*c*d - b*e) + B*(15
*c^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*
x^4]) + (a^(1/4)*(10*A*c*e*(3*c*d - b*e) + B*(15*c^2*d^2 + 8*b^2*e^2 - c*e*
(20*b*d + 9*a*e)) - (Sqrt[c]*(2*a*B*e*(5*c*d - 2*b*e) - 5*A*c*(3*c*d^2 - a*
e^2)))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] $\quad$ time = 0.726367, antiderivative size = 528, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1679, 1197, 1103, 1195}

$$\dfrac{x\sqrt{a+bx^2+cx^4}\left(B\left(-ce(9ae+20bd)+8b^2e^2+15c^2d^2\right)+10Ace(3cd-be)\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\dfrac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\dfrac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Int[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*(10*B*c*d - 4*b*B*e + 5*A*c*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*
e^2*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + ((10*A*c*e*(3*c*d - b*e) + B*(15*c
^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(15*
c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(10*A*c*e*(3*c*d - b*e) + B*(15
*c^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*
x^4]) + (a^(1/4)*(10*A*c*e*(3*c*d - b*e) + B*(15*c^2*d^2 + 8*b^2*e^2 - c*e*
(20*b*d + 9*a*e)) - (Sqrt[c]*(2*a*B*e*(5*c*d - 2*b*e) - 5*A*c*(3*c*d^2 - a*
e^2)))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
```

```
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^2}{\sqrt{a + bx^2 + cx^4}}\, dx = \frac{Be^2x^3\sqrt{a + bx^2 + cx^4}}{5c} + \frac{\int \frac{5Acd^2 + \left(5Bcd^2 + 10Acde - 3aBe^2\right)x^2 + e(10Bcd - 4bBe + 5Ace)x^4}{\sqrt{a+bx^2+cx^4}}\, dx}{5c}$$

$$= \frac{e(10Bcd - 4bBe + 5Ace)x\sqrt{a + bx^2 + cx^4}}{15c^2} + \frac{Be^2x^3\sqrt{a + bx^2 + cx^4}}{5c} + \frac{\int \frac{-2aBe(5cd-2be)+5Ac(3ca\ldots}{\ldots}}{\ldots}$$

$$= \frac{e(10Bcd - 4bBe + 5Ace)x\sqrt{a + bx^2 + cx^4}}{15c^2} + \frac{Be^2x^3\sqrt{a + bx^2 + cx^4}}{5c} - \frac{\left(\sqrt{a}\left(10Ace(3cd - be)\ldots\right.}{\ldots}$$

$$= \frac{e(10Bcd - 4bBe + 5Ace)x\sqrt{a + bx^2 + cx^4}}{15c^2} + \frac{Be^2x^3\sqrt{a + bx^2 + cx^4}}{5c} + \frac{\left(10Ace(3cd - be) + B\ldots\right.}{\ldots}$$

Mathematica [C]   time = 4.57953, size = 674, normalized size = 1.28

$$-i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right), \frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)\left(c\left(10Ac\left(e\left(3d\sqrt{b^2-4ac}-ae\right)+\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*x*(a + b*x^2 + c*x^4)*(5*A*c*e + B*(
10*c*d - 4*b*e + 3*c*e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*(10*A*c*e*(3*c*d
- b*e) + B*(15*c^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*Sqrt[(b + Sqrt[
```

```
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4
*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])] - I*(-8*b^3*B*e^2 + 2*b^2*e*(10*B*c*d + 5*A*c*e + 4*B*Sqrt[b^2 - 4*a*c
]*e) - b*c*(15*B*c*d^2 + B*e*(20*Sqrt[b^2 - 4*a*c]*d - 17*a*e) + 10*A*e*(3*
c*d + Sqrt[b^2 - 4*a*c]*e)) + c*(B*(-9*a*Sqrt[b^2 - 4*a*c]*e^2 + 5*c*d*(3*S
qrt[b^2 - 4*a*c]*d - 4*a*e)) + 10*A*c*(3*c*d^2 + e*(3*Sqrt[b^2 - 4*a*c]*d -
 a*e))))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])]])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
a + b*x^2 + c*x^4])
```

Maple [B]   time = 0.009, size = 1201, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)

[Out] B*e^2*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1
/15*b/c^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/
2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*a/c+8/15*b^2/c^2)*a*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4
+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^
2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(A*e^
2+2*B*d*e)*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*a/c*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+
1/3*b/c*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)
-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2
^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))))-1/2*(2*A*d*e+B*d^2)*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2
)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2
b(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/4*A*d^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a
c+b^2)^(1/2)-b)/a)^(1/2),1/2(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^2}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Be^2x^6 + \left(2\,Bde + Ae^2\right)x^4 + Ad^2 + \left(Bd^2 + 2\,Ade\right)x^2}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((B*e^2*x^6 + (2*B*d*e + A*e^2)*x^4 + A*d^2 + (B*d^2 + 2*A*d*e)*x^2
)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^2}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)*(d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^2}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.19 $\quad \int \dfrac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=368

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}+3Ace-2bBe+3Bcd\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+}}{}$$

```
[Out] (B*e*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + ((3*B*c*d - 2*b*B*e + 3*A*c*e)*x*Sq
rt[a + b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(3*B*
c*d - 2*b*B*e + 3*A*c*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(3*B*c
*d - 2*b*B*e + 3*A*c*e + (Sqrt[c]*(3*A*c*d - a*B*e))/Sqrt[a])*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[
a + b*x^2 + c*x^4])
```

Rubi [A] $\quad$ time = 0.25398, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1679, 1197, 1103, 1195}

$$\frac{x\sqrt{a+bx^2+cx^4}(3Ace-2bBe+3Bcd)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}\right.}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*e*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + ((3*B*c*d - 2*b*B*e + 3*A*c*e)*x*Sq
rt[a + b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(3*B*
c*d - 2*b*B*e + 3*A*c*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(3*B*c
*d - 2*b*B*e + 3*A*c*e + (Sqrt[c]*(3*A*c*d - a*B*e))/Sqrt[a])*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[
a + b*x^2 + c*x^4])
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)}{\sqrt{a + bx^2 + cx^4}} \, dx = \frac{Bex\sqrt{a + bx^2 + cx^4}}{3c} + \frac{\int \frac{3Acd - aBe + (3Bcd - 2bBe + 3Ace)x^2}{\sqrt{a + bx^2 + cx^4}} \, dx}{3c}$$

$$= \frac{Bex\sqrt{a + bx^2 + cx^4}}{3c} - \frac{\left(\sqrt{a}(3Bcd - 2bBe + 3Ace)\right)\int \frac{1 - \frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}} \, dx}{3c^{3/2}} + \frac{\left(\sqrt{a}\left(3Bcd - 2bBe + 3.\right.}{}$$

$$= \frac{Bex\sqrt{a + bx^2 + cx^4}}{3c} + \frac{(3Bcd - 2bBe + 3Ace)x\sqrt{a + bx^2 + cx^4}}{3c^{3/2}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(3Bcd - 2bBe + 3Ace)\left(\right.}{}$$

Mathematica [C]   time = 2.23374, size = 521, normalized size = 1.42

$$i\sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}}\sqrt{\frac{-2\sqrt{b^2 - 4ac} + 2b + 4cx^2}{b - \sqrt{b^2 - 4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}}\right), \frac{\sqrt{b^2 - 4ac} + b}{b - \sqrt{b^2 - 4ac}}\right)\left(b\left(2Be\sqrt{b^2 - 4ac} + 3Ace + 3Bcd\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*x*(a + b*x^2 + c*x^4) - I*(-b + Sq
rt[b^2 - 4*a*c])*(-3*B*c*d + 2*b*B*e - 3*A*c*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2
*b^2*B*e - c*(6*A*c*d + 3*B*Sqrt[b^2 - 4*a*c]*d - 2*a*B*e + 3*A*Sqrt[b^2 -
4*a*c]*e) + b*(3*B*c*d + 3*A*c*e + 2*B*Sqrt[b^2 - 4*a*c]*e))*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B]   time = 0.007, size = 759, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] B*e*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*a/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^
2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c
*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*
(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))))-1/2*(A*e+B*d)*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a
*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c
+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2)))+1/4*A*d*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*
(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bex^4+(Bd+Ae)x^2+Ad}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.20 $\qquad \int \dfrac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(}{c^{3/4}\sqrt{a+bx}}$$

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A]   time = 0.0808877, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\, dx = \left(A + \frac{\sqrt{a}B}{\sqrt{c}}\right) \int \frac{1}{\sqrt{a + bx^2 + cx^4}}\, dx - \frac{\left(\sqrt{a}B\right) \int \frac{1 - \frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}}\, dx}{\sqrt{c}}$$

$$= \frac{Bx\sqrt{a + bx^2 + cx^4}}{\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a + bx^2 + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}} E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\mid \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a + bx^2 + cx^4}} + \frac{\left(\sqrt{a}B + $$

Mathematica [C]   time = 0.264608, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}}\sqrt{\frac{2cx^2}{b - \sqrt{b^2 - 4ac}} + 1}\left(\left(-B\sqrt{b^2 - 4ac} - 2Ac + bB\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}}\right), \frac{\sqrt{b^2 - 4ac} + b}{b - \sqrt{b^2 - 4ac}}\right) + B\left(\sqrt{b^2}\right)}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}}\sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]))
]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]   time = 0.006, size = 362, normalized size = 1.3

$$-\frac{Ba\sqrt{2}}{2}\sqrt{4 - 2\frac{\left(\sqrt{-4ac + b^2} - b\right)x^2}{a}}\sqrt{4 + 2\frac{\left(b + \sqrt{-4ac + b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(\sqrt{-4ac + b^2} - b\right)}, \frac{1}{2}\sqrt{-4 + 2\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*B*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-
b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2)))+1/4*A*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^
2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(
```

```
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.21 $\int \dfrac{A+Bx^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=436

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}\right.}{4\sqrt[4]{c}}$$

```
[Out] -((B*d - A*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[
a + b*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2]) - ((S
qrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^
2 + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[
-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*e*(c*d^2 - a*e^2)*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi [A]   time = 0.408582, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1708, 1103, 1706}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\mid\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}\left(cd^2-ae^2\right)}-\frac{(Bd-Ae)\tan^{-1}\left(\frac{x\sqrt{}}{\sqrt{d}\sqrt{}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bd}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((B*d - A*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[
a + b*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2]) - ((S
qrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^
2 + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[
-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*e*(c*d^2 - a*e^2)*Sqrt[a
 + b*x^2 + c*x^4])
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

<u>Rule 1706</u>

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

<u>Rubi steps</u>

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\, dx = -\frac{\left(\sqrt{a}B - A\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\, dx}{\sqrt{c}d - \sqrt{a}e} + \frac{\left(\sqrt{a}(Bd - Ae)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\, dx}{\sqrt{c}d - \sqrt{a}e}$$

$$= -\frac{(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2 - bde + ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2 - bde + ae^2}} - \frac{\left(\sqrt{a}B - A\sqrt{c}\right)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\right.}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d - \sqrt{a}e\right)\sqrt{a + }}$$

Mathematica [C]   time = 0.513092, size = 298, normalized size = 0.68

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}} + 1}\left(Bd\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right) + (Ae - Bd)\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\right.\right.}{\sqrt{2}de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*d*EllipticF[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])] + (-(B*d) + A*e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*A
rcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [A]   time = 0.029, size = 359, normalized size = 0.8

$$\frac{B\sqrt{2}}{4e}\sqrt{4 - 2\frac{\left(\sqrt{-4ac + b^2} - b\right)x^2}{a}}\sqrt{4 + 2\frac{\left(b + \sqrt{-4ac + b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(\sqrt{-4ac + b^2} - b\right)}, \frac{1}{2}\sqrt{-4 + 2\frac{b}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B/e*2^(1/2)/((((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(A*e-B*d)/e/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/
2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a
*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi
(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a
*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/
a)^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.22 $\displaystyle\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=782

$$\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{ae}\right)}+\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2-bde+cd^2\right)}-\frac{ex\sqrt{a+}}{2d\left(d+e\right)}$$

```
[Out] (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2
)*(Sqrt[a] + Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d
*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) - ((B*(c*d^3 - a*d*e^2) - A*e*(3*c*d^
2 - e*(2*b*d - a*e)))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[
e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2)*Sqrt[e]*(c*d^2 - b*d*e + a*e^2)^(3
/2)) - (a^(1/4)*c^(1/4)*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^
2 + c*x^4]) + (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 +
 c*x^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*(c*d^3 - a*d*e^2) - A*e*(3*c*d^2 - e
*(2*b*d - a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*
d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/
4)*c^(1/4)*d^2*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x^2 + c*x^4])
```

Rubi [A]   time = 1.4669, antiderivative size = 782, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1696, 1714, 1195, 1708, 1103, 1706}

$$\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2-bde+cd^2\right)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\left(B\left(cd^3-ade^2\right)-Ae\left(3cd^2-e(2bd-ae)\right)\right)\tan^{-1}\left(\frac{-}{-}\right)}{4d^{3/2}\sqrt{e}\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2
)*(Sqrt[a] + Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d
*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) - ((B*(c*d^3 - a*d*e^2) - A*e*(3*c*d^
2 - e*(2*b*d - a*e)))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[
e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2)*Sqrt[e]*(c*d^2 - b*d*e + a*e^2)^(3
/2)) - (a^(1/4)*c^(1/4)*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^
2 + c*x^4]) + (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 +
 c*x^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*(c*d^3 - a*d*e^2) - A*e*(3*c*d^2 - e
*(2*b*d - a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*
d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/
```

4)*c^(1/4)*d^2*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x^2 + c*x^4])

Rule 1696

Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1714

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c,
 0]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

Rule 1708

Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1706

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a

```
+ b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))]], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^2 \sqrt{a + bx^2 + cx^4}}\, dx = -\frac{e(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{2d\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)} - \frac{\int \frac{-aBde - A\left(2cd^2 - e(2bd - ae)\right) - 2cd(Bd - Ae)x^2 - ce(Bd - Ae)x^4}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\, dx}{2d\left(cd^2 - bde + ae^2\right)}$$

$$= -\frac{e(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{2d\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)} - \frac{\int \frac{-\sqrt{a}c^{3/2}de(Bd - Ae) + ce\left(-aBde - A\left(2cd^2 - e(2bd - ae)\right)\right) + \left(-2c^2de(Bd\right.}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}}{2cde\left(cd^2 - bde + ae^2\right)}$$

$$= \frac{\sqrt{c}(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{2d\left(cd^2 - bde + ae^2\right)\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{2d\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd - Ae)}{}$$

$$= \frac{\sqrt{c}(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{2d\left(cd^2 - bde + ae^2\right)\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{2d\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)} - \frac{\left(B\left(cd^3 - ade^2\right)\right.}{}$$

Mathematica [C]   time = 3.82199, size = 1853, normalized size = 2.37

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I/8)*((-4*I)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*(B*d - A*e)*x*(a + b*
x^2 + c*x^4) + Sqrt[2]*B*(b - Sqrt[b^2 - 4*a*c])*d^2*e*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])]) + Sqrt[2]*A*(-b + Sqrt[b^2 - 4*a*c])*d*e^2*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + 2*Sqrt[2]*B*c*d^3*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
 - 2*Sqrt[2]*A*c*d^2*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*Ellipti
cF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] - 2*Sqrt[2]*B*c*d^3*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])] + 6*Sqrt[2]*A*c*d^2*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
```

```
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])] - 4*Sqrt[2]*A*b*d*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d +
e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])] + 2*Sqrt[2]*a*B*d*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*
EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
 + 2*Sqrt[2]*a*A*e^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticP
i[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]]))/(Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*d*e*(c*d^3 + d*e*(-(b*d) + a*e))*(d + e*x^2)*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [B]   time = 0.027, size = 1495, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] (A*e-B*d)/e*(1/2*e^2/(a*e^2-b*d*e+c*d^2)/d*x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d
)-1/8*c/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4
-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^
2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e
/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/
a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)
^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*
2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-1/4*c*e/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2
)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+
2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/
2))*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(1
/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*
x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*
a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)*e/d*2^(1/2)/(1/a*(-4*a
*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/
2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)
*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)
^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)/a)^(1/2))*b+3/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1
/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/
a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticP
i(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*
a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2))*c)+B/e/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x
^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b
```

```
^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^2 \sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.23 $\quad \displaystyle\int \frac{A+Bx^2}{\left(d+ex^2\right)^3 \sqrt{a+bx^2+cx^4}}\, dx$

Optimal. Leaf size=1125

<div align="center">result too large to display</div>

```
[Out] -(Sqrt[c]*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*d*(5*c*d^2 - e*(2*b*d + a*
e)))*x*Sqrt[a + b*x^2 + c*x^4])/(8*d^2*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] +
 Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(4*d*(c*d^2 - b*
d*e + a*e^2)*(d + e*x^2)^2) + (e*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*d*(
5*c*d^2 - e*(2*b*d + a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(8*d^2*(c*d^2 - b*d*
e + a*e^2)^2*(d + e*x^2)) - ((B*d*(3*c^2*d^4 - 10*a*c*d^2*e^2 + a*e^3*(4*b*
d - a*e)) - A*e*(15*c^2*d^4 - 2*c*d^2*e*(10*b*d - 3*a*e) + e^2*(8*b^2*d^2 -
 8*a*b*d*e + 3*a^2*e^2)))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*S
qrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(16*d^(5/2)*Sqrt[e]*(c*d^2 - b*d*e + a*e^
2)^(5/2)) + (a^(1/4)*c^(1/4)*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*d*(5*c*
d^2 - e*(2*b*d + a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(8*d^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^
4]) + (c^(1/4)*(Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(B*d + 3*A*e) + 4*A*d*(
c*d - b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqr
t[a + b*x^2 + c*x^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*d*(3*c^2*d^4 - 10*a*c*d
^2*e^2 + a*e^3*(4*b*d - a*e)) - A*e*(15*c^2*d^4 - 2*c*d^2*e*(10*b*d - 3*a*e
) + e^2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqr
t[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(32*a^(1/4)*c^(1/4)*d^3*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 3.59431, antiderivative size = 1125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1696, 1714, 1195, 1708, 1103, 1706}

$$-\frac{\sqrt{c}\left(3Ae\left(3cd^2 - e(2bd - ae)\right) - B\left(5cd^3 - de(2bd + ae)\right)\right)\sqrt{cx^4 + bx^2 + ax}}{8d^2\left(cd^2 - bed + ae^2\right)^2\left(\sqrt{c}x^2 + \sqrt{a}\right)} + \frac{e\left(3Ae\left(3cd^2 - e(2bd - ae)\right) - B\left(5cd^3 - \right.}{8d^2\left(cd^2 - bed + ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(5*c*d^3 - d*e*(2*b*d + a*
e)))*x*Sqrt[a + b*x^2 + c*x^4])/(8*d^2*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] +
 Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(4*d*(c*d^2 - b*
d*e + a*e^2)*(d + e*x^2)^2) + (e*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(5*
c*d^3 - d*e*(2*b*d + a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(8*d^2*(c*d^2 - b*d*
e + a*e^2)^2*(d + e*x^2)) - ((B*(3*c^2*d^5 - 10*a*c*d^3*e^2 + a*d*e^3*(4*b*
d - a*e)) - A*e*(15*c^2*d^4 - 2*c*d^2*e*(10*b*d - 3*a*e) + e^2*(8*b^2*d^2 -
 8*a*b*d*e + 3*a^2*e^2)))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*S
qrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(16*d^(5/2)*Sqrt[e]*(c*d^2 - b*d*e + a*e^
2)^(5/2)) + (a^(1/4)*c^(1/4)*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(5*c*d^
3 - d*e*(2*b*d + a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
```

qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4)])/(8*d^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^
4]) + (c^(1/4)*(Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(B*d + 3*A*e) + 4*A*d*(
c*d - b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4)])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqr
t[a + b*x^2 + c*x^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*(3*c^2*d^5 - 10*a*c*d^3
*e^2 + a*d*e^3*(4*b*d - a*e)) - A*e*(15*c^2*d^4 - 2*c*d^2*e*(10*b*d - 3*a*e
) + e^2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqr
t[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4)])/(32*a^(1/4)*c^(1/4)*d^3*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4])

Rule 1696

Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - B*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]

Rule 1714

Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && !GtQ[b^2 - 4*a*c,
 0]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

Rule 1708

Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^3 \sqrt{a + bx^2 + cx^4}}\, dx = -\frac{e(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{4d\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)^2} - \frac{\int \frac{-4Acd^2 - aBde + Ae(4bd - 3ae) - 2(Bd - Ae)(2cd - be)x^2 + ce(Bd -}{\left(d + ex^2\right)^2 \sqrt{a + bx^2 + cx^4}}}{4d\left(cd^2 - bde + ae^2\right)}$$

$$= -\frac{e(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{4d\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)^2} + \frac{e\left(3Ae\left(3cd^2 - e(2bd - ae)\right) - B\left(5cd^3 - de(2bd}{8d^2\left(cd^2 - bde + ae^2\right)^2\left(d +}$$

$$= -\frac{e(Bd - Ae)x\sqrt{a + bx^2 + cx^4}}{4d\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)^2} + \frac{e\left(3Ae\left(3cd^2 - e(2bd - ae)\right) - B\left(5cd^3 - de(2bd}{8d^2\left(cd^2 - bde + ae^2\right)^2\left(d +}$$

$$= -\frac{\sqrt{c}\left(3Ae\left(3cd^2 - e(2bd - ae)\right) - B\left(5cd^3 - de(2bd + ae)\right)\right)x\sqrt{a + bx^2 + cx^4}}{8d^2\left(cd^2 - bde + ae^2\right)^2\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(B}{4d\left(}$$

$$= -\frac{\sqrt{c}\left(3Ae\left(3cd^2 - e(2bd - ae)\right) - B\left(5cd^3 - de(2bd + ae)\right)\right)x\sqrt{a + bx^2 + cx^4}}{8d^2\left(cd^2 - bde + ae^2\right)^2\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{e(B}{4d\left(}$$

Mathematica [C]   time = 6.1701, size = 781, normalized size = 0.69

$$-\frac{4de^2x\left(a + bx^2 + cx^4\right)\left(\left(d + ex^2\right)\left(B\left(5cd^3 - de(ae + 2bd)\right) - 3Ae\left(e(ae - 2bd) + 3cd^2\right)\right) + 2d(Bd - Ae)\left(e(ae - bd) + cd^2\right)\right)}{\left(d + ex^2\right)^2} - \frac{i\sqrt{2}\sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}}\sqrt{\frac{2cx^2}{b - \sqrt{b^2 - 4ac}} + 1}\left(d\text{Elliptic}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ((-4*d*e^2*x*(a + b*x^2 + c*x^4)*(2*d*(B*d - A*e)*(c*d^2 + e*(-(b*d) + a*e)
) + (-3*A*e*(3*c*d^2 + e*(-2*b*d + a*e)) + B*(5*c*d^3 - d*e*(2*b*d + a*e)))
*(d + e*x^2)))/(d + e*x^2)^2 - (I*Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
```

```
((-b + Sqrt[b^2 - 4*a*c])*d*e*(3*A*e*(3*c*d^2 + e*(-2*b*d + a*e)) + B*(-5*c
*d^3 + d*e*(2*b*d + a*e)))*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + d*(B*d*(
6*c^2*d^3 + c*d*e*(-5*b*d + 5*Sqrt[b^2 - 4*a*c]*d - 6*a*e) - (-b + Sqrt[b^2
 - 4*a*c])*e^2*(2*b*d + a*e)) - A*e*(14*c^2*d^3 - 3*(-b + Sqrt[b^2 - 4*a*c]
)*e^2*(2*b*d - a*e) + c*d*e*(-17*b*d + 9*Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*Ell
ipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + 2*(B*(-3*c^2*d^5 + 10*a*c*d^3*e^2 + a*
d*e^3*(-4*b*d + a*e)) + A*e*(15*c^2*d^4 + 2*c*d^2*e*(-10*b*d + 3*a*e) + e^2
*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)))*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]])/
(32*d^3*e*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B]   time = 0.031, size = 4476, normalized size = 4.

<div align="center">output too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] B/e*(1/2*e^2/(a*e^2-b*d*e+c*d^2)/d*x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)-1/8*c/
(a*e^2-b*d*e+c*d^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2
*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2
))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)
-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/(a*e^2-
b*d*e+c*d^2)/d*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-
4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^
(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*
(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-1/4*c*e/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)
^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*
(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*Elli
pticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(1/a*(-4*a
*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/
2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)
*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)
^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)*e/d*2^(1/2)/(1/a*(-4*a*c+b^2)^
(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/
2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipti
cPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b
)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)/a)^(1/2))*b+3/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*
b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+
1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*
2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-
1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2
))*c)+(A*e-B*d)/e*(1/4*e^2/(a*e^2-b*d*e+c*d^2)/d*x*(c*x^4+b*x^2+a)^(1/2)/(e
*x^2+d)^2+3/8*e^2*(a*e^2-2*b*d*e+3*c*d^2)/(a*e^2-b*d*e+c*d^2)^2/d^2*x*(c*x^
4+b*x^2+a)^(1/2)/(e*x^2+d)+3/16*e^3*c/(a*e^2-b*d*e+c*d^2)^2/d^2*a^2*2^(1/2)
/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x
^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+9/16*e*c^2/(a*e^2-b
```

```
*d*e+c*d^2)^2*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4
*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*(
((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))-3/8*e^2*c/(a*e^2-b*d*e+c*d^2)^2/d*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a
*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x
^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*E
llipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))*b+3/8*e^2*c/(a*e^2-b*d*e+c*d^2)^2/d*a*2^(1/2)/(
1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2
)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b-7/32*c^2/(a*e^2-b*d
*e+c*d^2)^2*d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a
*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/8*c/(a*e^2-b*d*e+c*
d^2)^2*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)
^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b*e-1/(a*e^2-b*d*e+c*d^2)^2/
d^2*e^3*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b
^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*a*b+3/4/(a*e^2-b*d*e+c*d^2)^
2*e^2/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b
^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*a*c-1/32*c/(a*e^2-b*d*e+c*d^
2)^2/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)
^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*a*e^2+3/8/(a*e^2-b*d*e+c*d^2
)^2/d^3*e^4*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a
*c+b^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2
))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2
)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*a^2+1/(a*e^2-b*d*e+c*d^2
)^2*e^2/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c
+b^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-
b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*b^2-5/2/(a*e^2-b*d*e+c*d^2
)^2*e*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2
)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a
)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*b*c+15/8/(a*e^2-b*d*e+c*d^2)^2
*d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(
1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(
1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*c^2-3/16*e^3*c/(a*e^2-b*d*e+c*d^2
)^2/d^2*a^2*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c
+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
```

```
9/16*e*c^2/(a*e^2-b*d*e+c*d^2)^2*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(
1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-
4*a*c+b^2)^(1/2)))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*Ellipt
icE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^3), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^3 \sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/((d + e*x**2)**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^3), x)
```

3.24 $\displaystyle\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=859

$$\frac{Bx\sqrt{cx^4+bx^2+ae^3}}{3c^2} - \frac{\left(aB\left(6c^3d^3 - 9c^2e(bd+6ae)d - 8b^3e^3 + bce^2(18bd+29ae)\right) + 3Ac\left(2ab^2e^3 + 6ac\left(cd^2-ae^2\right)e - \cdots\right)}{3a^{3/4}c^{11/4}\left(b^2-4ac\right)\sqrt{cx^4 \cdots}}$$

```
[Out] (x*(A*c*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2)) +
 a*B*(a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)) - (a
*B*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)) + A*c*(a*b^2*e^3 +
 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)))*x^2))/(a*c^2*(b^2 -
4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (B*e^3*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^2)
 + ((a*B*(6*c^3*d^3 - 8*b^3*e^3 - 9*c^2*d*e*(b*d + 6*a*e) + b*c*e^2*(18*b*d
 + 29*a*e)) + 3*A*c*(2*a*b^2*e^3 + 6*a*c*e*(c*d^2 - a*e^2) - b*c*d*(c*d^2 +
 3*a*e^2)))*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*c^(5/2)*(b^2 - 4*a*c)*(Sqrt[a]
+ Sqrt[c]*x^2)) - ((a*B*(6*c^3*d^3 - 8*b^3*e^3 - 9*c^2*d*e*(b*d + 6*a*e) +
b*c*e^2*(18*b*d + 29*a*e)) + 3*A*c*(2*a*b^2*e^3 + 6*a*c*e*(c*d^2 - a*e^2) -
 b*c*d*(c*d^2 + 3*a*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(3/4)*c^(11/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 +
c*x^4]) - ((3*A*c^3*d^3 - 5*a^2*B*c*e^3 - 3*Sqrt[a]*c^(5/2)*d^2*(B*d + 3*A*
e) + a*e*(3*c*d - 2*b*e)*(3*B*c*d - 4*b*B*e + 3*A*c*e) + 3*a^(3/2)*Sqrt[c]*
e^2*(9*B*c*d - 4*b*B*e + 3*A*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(11/4)
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A]   time = 1.37963, antiderivative size = 859, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1678, 1679, 1197, 1103, 1195}

$$\frac{Bx\sqrt{cx^4+bx^2+ae^3}}{3c^2} - \frac{\left(aB\left(6c^3d^3 - 9c^2e(bd+6ae)d - 8b^3e^3 + bce^2(18bd+29ae)\right) + 3Ac\left(2ab^2e^3 + 6ac\left(cd^2-ae^2\right)e - \cdots\right)}{3a^{3/4}c^{11/4}\left(b^2-4ac\right)\sqrt{cx^4 \cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*c*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2)) +
 a*B*(a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)) - (a
*B*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)) + A*c*(a*b^2*e^3 +
 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)))*x^2))/(a*c^2*(b^2 -
4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (B*e^3*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^2)
 + ((a*B*(6*c^3*d^3 - 8*b^3*e^3 - 9*c^2*d*e*(b*d + 6*a*e) + b*c*e^2*(18*b*d
 + 29*a*e)) + 3*A*c*(2*a*b^2*e^3 + 6*a*c*e*(c*d^2 - a*e^2) - b*c*d*(c*d^2 +
 3*a*e^2)))*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*c^(5/2)*(b^2 - 4*a*c)*(Sqrt[a]
+ Sqrt[c]*x^2)) - ((a*B*(6*c^3*d^3 - 8*b^3*e^3 - 9*c^2*d*e*(b*d + 6*a*e) +
b*c*e^2*(18*b*d + 29*a*e)) + 3*A*c*(2*a*b^2*e^3 + 6*a*c*e*(c*d^2 - a*e^2) -
 b*c*d*(c*d^2 + 3*a*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(3/4)*c^(11/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 +
```

c*x^4]) - ((3*A*c^3*d^3 - 5*a^2*B*c*e^3 - 3*Sqrt[a]*c^(5/2)*d^2*(B*d + 3*A*
e) + a*e*(3*c*d - 2*b*e)*(3*B*c*d - 4*b*B*e + 3*A*c*e) + 3*a^(3/2)*Sqrt[c]*
e^2*(9*B*c*d - 4*b*B*e + 3*A*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(11/4)
*Sqrt[a + b*x^2 + c*x^4])

Rule 1678

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1679

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^3}{\left(a + bx^2 + cx^4\right)^{3/2}}\,dx = \frac{x\left(Ac\left(b^2cd^3 - 2acd\left(cd^2 - 3ae^2\right) - abe\left(3cd^2 + ae^2\right)\right) + aB\left(ab^2e^3 + 2ace\left(3cd^2 - ae^2\right) - \right.}{}$$

$$= \frac{x\left(Ac\left(b^2cd^3 - 2acd\left(cd^2 - 3ae^2\right) - abe\left(3cd^2 + ae^2\right)\right) + aB\left(ab^2e^3 + 2ace\left(3cd^2 - ae^2\right) - \right.}{}$$

$$= \frac{x\left(Ac\left(b^2cd^3 - 2acd\left(cd^2 - 3ae^2\right) - abe\left(3cd^2 + ae^2\right)\right) + aB\left(ab^2e^3 + 2ace\left(3cd^2 - ae^2\right) - \right.}{}$$

$$= \frac{x\left(Ac\left(b^2cd^3 - 2acd\left(cd^2 - 3ae^2\right) - abe\left(3cd^2 + ae^2\right)\right) + aB\left(ab^2e^3 + 2ace\left(3cd^2 - ae^2\right) - \right.}{}$$

Mathematica [C]   time = 6.67421, size = 5432, normalized size = 6.32

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x]

[Out] Result too large to show
```

Maple [B]   time = 0.034, size = 2445, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x)

[Out] B*e^3*(-2*c*(-1/2*b/c^3*(3*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)*a/c^3/(
4*a*c-b^2)*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)+1/3/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1
/4*(-1/c^2*(2*a*c-b^2)*a/(4*a*c-b^2)-1/3*a/c^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*
c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2
*(-5/3*b/c^2-b/c^2*(3*a*c-b^2)/(4*a*c-b^2))*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(
1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(A*e^3+3*B*d*e^2)*(-2*c*(
1/2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*b/c^2/(4*a*c-b^2)*x)/((x^4+x^2*b/
c+a/c)*c)^(1/2)-1/4/c*a*b/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1
/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/
2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(1/c+1/c*
(2*a*c-b^2)/(4*a*c-b^2))*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((
```

```
-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))))+(3*A*d*e^2+3*B*d^2*e)*(-2*c*(1/2*b/(4*a*c-b^
2)/c*x^3+a/c/(4*a*c-b^2)*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)+1/2*a/(4*a*c-b^2)*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^
(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipt
icF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/
a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(3*A*d^2*e+B*d^3)*(-2*c*(-1/
(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)-1/4*b/(4
*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)
-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellipt
icF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*d^3*(-2*c*(1/2/a*b/
(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/c)*c)^(1
/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^
(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(
1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*
c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-
b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Be^3x^8 + \left(3\,Bde^2 + Ae^3\right)x^6 + 3\left(Bd^2e + Ade^2\right)x^4 + Ad^3 + \left(Bd^3 + 3\,Ad^2e\right)x^2\right)\sqrt{cx^4 + bx^2 + a}}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out]  integral((B*e^3*x^8 + (3*B*d*e^2 + A*e^3)*x^6 + 3*(B*d^2*e + A*d*e^2)*x^4 +
 A*d^3 + (B*d^3 + 3*A*d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a)/(c^2*x^8 + 2*b*c*
x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out]  Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^3}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out]  integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.25 $\int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=628

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(3a^{3/2}B\sqrt{c}e^2+ae(Ace-2bBe+2Bcd)-\sqrt{a}c^{3/2}d(2A}{2a^{3/4}c^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] -((x*(a*B*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - A*c*(b^2*d^2 - 2*a*b*d*e - 2*a*
(c*d^2 - a*e^2)) - (A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - a*B*(2*c^2*d^2 +
b^2*e^2 - 2*c*e*(b*d + a*e)))*x^2))/(a*c*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x
^4])) - ((A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - 2*a*B*(c^2*d^2 + b^2*e^2 -
c*e*(b*d + 3*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(a*c^(3/2)*(b^2 - 4*a*c)*(Sq
rt[a] + Sqrt[c]*x^2)) + ((A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - 2*a*B*(c^2*
d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*
x^2 + c*x^4]) - ((A*c^2*d^2 + 3*a^(3/2)*B*Sqrt[c]*e^2 - Sqrt[a]*c^(3/2)*d*(
B*d + 2*A*e) + a*e*(2*B*c*d - 2*b*B*e + A*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt
[c])*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A]   time = 0.584427, antiderivative size = 633, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1678, 1197, 1103, 1195}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(Ac\left(abe^2-4acde+bcd^2\right)-2aB\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)}{a^{3/4}c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(c*((a*B*(b*c*d^2 - 4*a*c*d*e + a*b*e^2))/c - A*(b^2*d^2 - 2*a*b*d*e -
 2*a*(c*d^2 - a*e^2))) - (A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - a*B*(2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)))*x^2))/(a*c*(b^2 - 4*a*c)*Sqrt[a + b*x^2
 + c*x^4])) - ((A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - 2*a*B*(c^2*d^2 + b^2*
e^2 - c*e*(b*d + 3*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(a*c^(3/2)*(b^2 - 4*a*
c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - 2*a*B
*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*c^(7/4)*(b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4]) - ((A*c^2*d^2 + 3*a^(3/2)*B*Sqrt[c]*e^2 - Sqrt[a]*c^(3/
2)*d*(B*d + 2*A*e) + a*e*(2*B*c*d - 2*b*B*e + A*c*e))*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a
]*Sqrt[c])*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)^2}{\left(a+bx^2+cx^4\right)^{3/2}}\, dx = -\frac{x\left(c\left(\frac{aB\left(bcd^2-4acde+abe^2\right)}{c}-A\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)\right)\right)-\left(Ac\left(bcd^2-4acde+ab\right.\right.}{ac\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{x\left(c\left(\frac{aB\left(bcd^2-4acde+abe^2\right)}{c}-A\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)\right)\right)-\left(Ac\left(bcd^2-4acde+ab\right.\right.}{ac\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{x\left(c\left(\frac{aB\left(bcd^2-4acde+abe^2\right)}{c}-A\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)\right)\right)-\left(Ac\left(bcd^2-4acde+ab\right.\right.}{ac\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Mathematica [C]   time = 4.76351, size = 766, normalized size = 1.22

$$i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)\left(Ac\left(b^2\left(ae^2+cd^2\right)-b\sqrt{b^2-4a}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-(a*B*(a*b*e^2 + 2*c^2*d^2*x^2 + b
^2*e^2*x^2 + b*c*d*(d - 2*e*x^2) - 2*a*c*e*(2*d + e*x^2))) + A*c*(b^2*d^2 +
 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(d + 2*e*x^2))) -
 I*(-b + Sqrt[b^2 - 4*a*c])*(-(A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)) + 2*a*B
*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(2*a*B*(b
^2*(-b + Sqrt[b^2 - 4*a*c])*e^2 + c^2*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + c*e
*(b^2*d - b*Sqrt[b^2 - 4*a*c]*d + 4*a*b*e - 3*a*Sqrt[b^2 - 4*a*c]*e)) + A*c
*(b^2*(c*d^2 + a*e^2) - b*Sqrt[b^2 - 4*a*c]*(c*d^2 + a*e^2) - 4*a*c*(c*d^2
- Sqrt[b^2 - 4*a*c]*d*e + a*e^2)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a*c^2*(-b^2 + 4*
a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B]   time = 0.008, size = 1891, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] B*e^2*(-2*c*(1/2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*b/c^2/(4*a*c-b^2)*x)
/((x^4+x^2*b/c+a/c)*c)^(1/2)-1/4/c*a*b/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
1/2*(1/c+1/c*(2*a*c-b^2)/(4*a*c-b^2))*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^
(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(A*e^2+2*B*d*e)*(-2*c*(1/2*b/(4
*a*c-b^2)/c*x^3+a/c/(4*a*c-b^2)*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)+1/2*a/(4*a*c
-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellipt
icF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(2*A*d*e+B*d^2)*(-2*c
*(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)-1/4
*b/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^
(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(E
llipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*d^2*(-2*c*(1/2
/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/c)*
```

```
c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b
^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/
(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(
1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/
2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2
*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^2}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(Be^2x^6 + \left(2\,Bde + Ae^2\right)x^4 + Ad^2 + \left(Bd^2 + 2\,Ade\right)x^2\right)\sqrt{cx^4 + bx^2 + a}}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((B*e^2*x^6 + (2*B*d*e + A*e^2)*x^4 + A*d^2 + (B*d^2 + 2*A*d*e)*x^2
)*sqrt(c*x^4 + b*x^2 + a)/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*
x^2 + a^2), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^2}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)*(d + e*x**2)**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^2}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.26 $\int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=481

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}c^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+b}{\left(\sqrt{a}+}}}{}$$

```
[Out] -((x*(a*B*(b*d - 2*a*e) - A*(b^2*d - 2*a*c*d - a*b*e) - (A*c*(b*d - 2*a*e)
- a*B*(2*c*d - b*e))*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) - ((A
*c*(b*d - 2*a*e) - a*B*(2*c*d - b*e))*x*Sqrt[a + b*x^2 + c*x^4])/(a*Sqrt[c]
*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*c*(b*d - 2*a*e) - a*B*(2*c*d
- b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(a^(3/4)*c^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B
- A*Sqrt[c])*(Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(3/4
)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A]   time = 0.387262, antiderivative size = 481, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1678, 1197, 1103, 1195}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(Ac(bd-2ae)-aB(2cd-be)\right)}{a^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(a*B*(b*d - 2*a*e) - A*(b^2*d - 2*a*c*d - a*b*e) - (A*c*(b*d - 2*a*e)
- a*B*(2*c*d - b*e))*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) - ((A
*c*(b*d - 2*a*e) - a*B*(2*c*d - b*e))*x*Sqrt[a + b*x^2 + c*x^4])/(a*Sqrt[c]
*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*c*(b*d - 2*a*e) - a*B*(2*c*d
- b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(a^(3/4)*c^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B
- A*Sqrt[c])*(Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(3/4
)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
```

```
   + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)}{\left(a + bx^2 + cx^4\right)^{3/2}}\, dx = -\frac{x\left(aB(bd - 2ae) - A\left(b^2d - 2acd - abe\right) - (Ac(bd - 2ae) - aB(2cd - be))x^2\right)}{a\left(b^2 - 4ac\right)\sqrt{a + bx^2 + cx^4}} - \frac{\int \frac{-a(bBd - 2Ac}{}}{}$$

$$= -\frac{x\left(aB(bd - 2ae) - A\left(b^2d - 2acd - abe\right) - (Ac(bd - 2ae) - aB(2cd - be))x^2\right)}{a\left(b^2 - 4ac\right)\sqrt{a + bx^2 + cx^4}} + \frac{\left(\left(\sqrt{a}B - A\right)}{}$$

$$= -\frac{x\left(aB(bd - 2ae) - A\left(b^2d - 2acd - abe\right) - (Ac(bd - 2ae) - aB(2cd - be))x^2\right)}{a\left(b^2 - 4ac\right)\sqrt{a + bx^2 + cx^4}} - \frac{(Ac(bd - 2a}{a\sqrt{}}$$

Mathematica [F]   time = 0, size = 0, normalized size = 0.

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B]   time = 0.006, size = 1390, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] B*e*(-2*c*(1/2*b/(4*a*c-b^2)/c*x^3+a/c/(4*a*c-b^2)*x)/((x^4+x^2*b/c+a/c)*c)
^(1/2)+1/2*a/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4
*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c
*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c
+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(A
*e+B*d)*(-2*c*(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/c)
*c)^(1/2)-1/4*b/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*(
(-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b
^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+
b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*d
*(-2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+x^
2*b/c+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*
(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4
*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c
*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a
*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-El
lipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bex^4 + (Bd + Ae)x^2 + Ad\right)\sqrt{cx^4 + bx^2 + a}}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)/(c^2*x^8
 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.27 $\int \dfrac{A+Bx^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=398

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)(Ab-2aB)\sqrt{\frac{a+}{\left(\sqrt{a}\right.}}}{a^{3/4}\left(b^2-4a\right.}$$

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*x^2 + c*x^4]) - ((A*b - 2*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(
b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*b - 2*a*B)*c^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 -
 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*
Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A]   time = 0.195045, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1178, 1197, 1103, 1195}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)(Ab-2aB)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F}{2a^{3/4}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*x^2 + c*x^4]) - ((A*b - 2*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(
b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*b - 2*a*B)*c^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 -
 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*
Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(a + bx^2 + cx^4\right)^{3/2}} \, dx = \frac{x\left(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2\right)}{a\left(b^2 - 4ac\right)\sqrt{a + bx^2 + cx^4}} - \frac{\int \frac{-a(bB - 2Ac) + (Ab - 2aB)cx^2}{\sqrt{a + bx^2 + cx^4}} \, dx}{a\left(b^2 - 4ac\right)}$$

$$= \frac{x\left(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2\right)}{a\left(b^2 - 4ac\right)\sqrt{a + bx^2 + cx^4}} + \frac{\left((Ab - 2aB)\sqrt{c}\right)\int \frac{1 - \frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}} \, dx}{\sqrt{a}\left(b^2 - 4ac\right)} - \frac{\left((Ab - 2aB)\sqrt{c}}{}$$

$$= \frac{x\left(Ab^2 - abB - 2aAc + (Ab - 2aB)cx^2\right)}{a\left(b^2 - 4ac\right)\sqrt{a + bx^2 + cx^4}} - \frac{(Ab - 2aB)\sqrt{c}x\sqrt{a + bx^2 + cx^4}}{a\left(b^2 - 4ac\right)\left(\sqrt{a} + \sqrt{c}x^2\right)} + \frac{(Ab - 2aB)\sqrt[4]{c}\left(\sqrt{}}{}$$

Mathematica [F]   time = 0, size = 0, normalized size = 0.

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B]   time = 0.006, size = 931, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] B*(-2*c*(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/c)*c)^(1
/2)-1/4*b/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*
```

```
c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1
/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(
1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*(-2*c*(
1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/
c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*
c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2
*b/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2
)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x
^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^
(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(
1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(Bx^2 + A\right)}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(c^2*x^8 + 2*b*c*x^6 + (b^2 +
2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.28 $\quad \int \dfrac{A+Bx^2}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=867

$$\dfrac{a^{3/4}e(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\dfrac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\Pi\left(-\dfrac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\dfrac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\dfrac{1}{4}\left(2-\dfrac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\dfrac{\sqrt{c}d}{\sqrt{a}}+e\right)^2}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}-\dfrac{e^{3/2}(Bd-Ae)\tan}{2\sqrt{d}\left(cd^2-\right.}$$

```
[Out] -((x*(a*b*c*(B*d - A*e) - (b^2 - 2*a*c)*(A*c*d - A*b*e + a*B*e) + c*(a*B*(2
*c*d - b*e) - A*(b*c*d - b^2*e + 2*a*c*e))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (Sqrt[c]*(a*B*(2*c*d - b*e) - A*
(b*c*d - b^2*e + 2*a*c*e))*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(c*d
^2 - b*d*e + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*ArcTan[
(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])
/(2*Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^(3/2)) - (c^(1/4)*(a*B*(2*c*d - b*e) -
A*(b*c*d - b^2*e + 2*a*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sq
rt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B - A*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2 - b/(4*Sqrt[a]*Sqrt[c])])/(2*a^(3/4)*(b - 2*Sq
rt[a]*Sqrt[c])*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) + (a^(3/4)*
e*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]
*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 - b*d*e + a*e^2)*Sqrt[
a + b*x^2 + c*x^4])
```

Rubi [A]　　time = 1.42356, antiderivative size = 1045, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 7, integrand size = 33, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1720, 1178, 1197, 1103, 1195, 1216, 1706}

$$\dfrac{a^{3/4}e(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\dfrac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\Pi\left(-\dfrac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\dfrac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\Big|\dfrac{1}{4}\left(2-\dfrac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\dfrac{\sqrt{c}d}{\sqrt{a}}+e\right)^2}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}-\dfrac{e^{3/2}(Bd-Ae)\tan}{2\sqrt{d}\left(cd^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((x*(a*b*c*(B*d - A*e) - (b^2 - 2*a*c)*(A*c*d - A*b*e + a*B*e) + c*(a*B*(2
*c*d - b*e) - A*(b*c*d - b^2*e + 2*a*c*e))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (Sqrt[c]*(a*B*(2*c*d - b*e) - A*
(b*c*d - b^2*e + 2*a*c*e))*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(c*d
^2 - b*d*e + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*ArcTan[
(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])
/(2*Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^(3/2)) - (c^(1/4)*(a*B*(2*c*d - b*e) -
A*(b*c*d - b^2*e + 2*a*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sq
rt[a + b*x^2 + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a
```

]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(a*B*e - S
qrt[a]*Sqrt[c]*(B*d - A*e) + A*(c*d - b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])
*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (a^(3/4)*e*((Sqrt[c]*d)
/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[
a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*
x^4])

Rule 1720

Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
x^2)^q(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[p + 1/2] && IntegerQ[q]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]

Rule 1197

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

Rule 1216

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,

```
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\int \frac{A + Bx^2}{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{3/2}}\,dx = \int \left(\frac{Acd - Abe + aBe + c(Bd - Ae)x^2}{\left(cd^2 - bde + ae^2\right)\left(a + bx^2 + cx^4\right)^{3/2}} + \frac{e(-Bd + Ae)}{\left(cd^2 - bde + ae^2\right)\left(d + ex^2\right)\sqrt{a + bx^2 -}}\right.
$$

$$
= \frac{\int \frac{Acd - Abe + aBe + c(Bd - Ae)x^2}{\left(a + bx^2 + cx^4\right)^{3/2}}\,dx}{cd^2 - bde + ae^2} - \frac{(e(Bd - Ae))\int \frac{1}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\,dx}{cd^2 - bde + ae^2}
$$

$$
= -\frac{x\left(abc(Bd - Ae) - \left(b^2 - 2ac\right)(Acd - Abe + aBe) + c\left(aB(2cd - be) - A\left(bcd - b\right.\right.}{a\left(b^2 - 4ac\right)\left(cd^2 - bde + ae^2\right)\sqrt{a + bx^2 + cx^4}}
$$

$$
= -\frac{x\left(abc(Bd - Ae) - \left(b^2 - 2ac\right)(Acd - Abe + aBe) + c\left(aB(2cd - be) - A\left(bcd - b\right.\right.}{a\left(b^2 - 4ac\right)\left(cd^2 - bde + ae^2\right)\sqrt{a + bx^2 + cx^4}}
$$

$$
= -\frac{x\left(abc(Bd - Ae) - \left(b^2 - 2ac\right)(Acd - Abe + aBe) + c\left(aB(2cd - be) - A\left(bcd - b\right.\right.}{a\left(b^2 - 4ac\right)\left(cd^2 - bde + ae^2\right)\sqrt{a + bx^2 + cx^4}}
$$

Mathematica [C]   time = 5.24307, size = 1736, normalized size = 2.

$$
4A\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}dexb^3 + 4Ac\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}dex^3b^2 - 4Ac\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d^2xb^2 - 4aB\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}dexb^2 + 2iaAe^2\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-4*A*b^2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*x + 4*a*b*B*c*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*d^2*x + 8*a*A*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*x
+ 4*A*b^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x - 4*a*b^2*B*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*d*e*x - 12*a*A*b*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x +
 8*a^2*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x - 4*A*b*c^2*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*d^2*x^3 + 8*a*B*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*x
^3 + 4*A*b^2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x^3 - 4*a*b*B*c*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*d*e*x^3 - 8*a*A*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
```

```
*d*e*x^3 - I*(-b + Sqrt[b^2 - 4*a*c])*d*(a*B*(2*c*d - b*e) + A*(-(b*c*d) +
b^2*e - 2*a*c*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*d*(a*B*(b*(b - Sqrt[b^2 - 4*a*c])
*e + 2*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + A*(-(b^3*e) + b*c*(-(Sqrt[b^2 - 4
*a*c]*d) + 4*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - 2*a*c*(2*c*d + Sqrt[b
^2 - 4*a*c]*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (2*I)*a*b^2*B*d*e*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*
c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (8*I)*a^2*B*c*d*e*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4
*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (2*I)*a*A*b^2*e^2*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
- 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*
a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (8*I)*a^2*A*c*e^2*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b
^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 -
 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]])/(4*a*(-b^2 + 4*a*c)*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*d*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*
x^4])
```

Maple [B]   time = 0.026, size = 3241, normalized size = 3.7

<div align="center">output too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] B/e*(-2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4
+x^2*b/c+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/
2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*(
(-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-
4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(A*e-B*d)/e*(-2*c*(1/2*(2*a*c*e-b^2*e+b*c*d
)/a/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)*x^3+1/2*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*
c*d)/a/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)/c*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)-1/4
*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)
+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x
^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*b*e+1/4*2^(1
/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*
b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)
```

```
^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*c*d+3/4*2^(1/2)/(
1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2
)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)*b*c*e-1/2*2^
(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/
a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)*c^2*d
-1/4*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(
1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(4*a*c-b^2)/(a*e^2-b*d*e+c*d
^2)*b^3*e+1/4*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a
*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(4*a*c-b^2)/(a*e^2-
b*d*e+c*d^2)*b^2*c*d-c^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)/(1/a*(-4*a
*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(
4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*e*a+1/2*c/(a*e^2-b*d*e+c*d^2)/
(4*a*c-b^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c
+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*
b^2*e-1/2*c^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/
2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2
+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1
/2))*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b*d+c^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2
^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2
/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2
+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/
2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*e*a-1/2*c/(a*
e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(
4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+
b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/
2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))*b^2*e+1/2*c^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)/(1/a
*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*b*x^2)^(
1/2)*(4+2/a*b*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b*d+1/(a*e^2-b*d*e+c*d^2
)*e^2/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b
^2)^(1/2)+1/2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [F (-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F (-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

3.29 $\int \dfrac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=1301

<center>result too large to display</center>

```
[Out] (x*(a*b*c*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2)) + (b^2 - 2*a*c)*(a*B*e*(2
*c*d - b*e) + A*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e))) - c*(a*B*(2*c^2*d^
2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 -
 b*c*(c*d^2 - 3*a*e^2)))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*S
qrt[a + b*x^2 + c*x^4]) + (Sqrt[c]*(a*B*d*(-4*c^2*d^2 - 3*b^2*e^2 + 4*c*e*(
b*d + 2*a*e)) + A*(2*b^3*d*e^2 + 2*b*c*d*(c*d^2 - 3*a*e^2) - 4*a*c*e*(-2*c*
d^2 + a*e^2) + b^2*(-4*c*d^2*e + a*e^3)))*x*Sqrt[a + b*x^2 + c*x^4])/(2*a*(
-b^2 + 4*a*c)*d*(c*d^2 + e*(-(b*d) + a*e))^2*(Sqrt[a] + Sqrt[c]*x^2)) - (e^
3*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x^2)) + (e^(3/2)*(A*e*(7*c*d^2 - e*(4*b*d - a*e)) - B*d*(5*c*d^2 - e*(2
*b*d + a*e)))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[
a + b*x^2 + c*x^4])])/(4*d^(3/2)*(c*d^2 - b*d*e + a*e^2)^(5/2)) - (c^(1/4)*
(a*B*d*(4*c^2*d^2 + 3*b^2*e^2 - 4*c*e*(b*d + 2*a*e)) - A*(2*b^3*d*e^2 + 2*b
*c*d*(c*d^2 - 3*a*e^2) - 4*a*c*e*(-2*c*d^2 + a*e^2) + b^2*(-4*c*d^2*e + a*e
^3)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2 - b/(4*Sqrt[a]*Sqrt[c])
])/(2*a^(3/4)*(b^2 - 4*a*c)*d*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + b*x^2 +
 c*x^4]) + (c^(1/4)*(a*Sqrt[c]*e*(B*d - 2*A*e) + Sqrt[a]*(B*d - A*e)*(c*d -
 b*e) + A*Sqrt[c]*d*(-(c*d) + b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2 - b/(4*Sqrt[a]*Sqrt[c])])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*d*(-(Sq
rt[c]*d) + Sqrt[a]*e)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) -
 (e*(Sqrt[c]*d + Sqrt[a]*e)*(A*e*(7*c*d^2 - e*(4*b*d - a*e)) - B*d*(5*c*d^2
 - e*(2*b*d + a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt
[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a
^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi [A] time = 3.38621, antiderivative size = 2112, normalized size of antiderivative = 1.62, number of steps used = 15, number of rules used = 10, integrand size = 33, $\dfrac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {1720, 1178, 1197, 1103, 1195, 1223, 1714, 1708, 1706, 1216}

<center>result too large to display</center>

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(a*b*c*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2)) + (b^2 - 2*a*c)*(a*B*e*(2
*c*d - b*e) + A*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e))) - c*(a*B*(2*c^2*d^
2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 -
 b*c*(c*d^2 - 3*a*e^2)))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*S
qrt[a + b*x^2 + c*x^4]) + (Sqrt[c]*e^2*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4
])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*(a*B*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e -
b^3*e^2 - b*c*(c*d^2 - 3*a*e^2)))*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*
c)*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A*e)*x*
Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) - (e^(
```

```
3/2)*(B*d - A*e)*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^2 - b*d*e + a
*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2)*(c*d^2 - b*
d*e + a*e^2)^(5/2)) + (e^(3/2)*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2))*ArcT
an[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4]
)])/(2*Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^(5/2)) - (a^(1/4)*c^(1/4)*e^2*(B*d -
 A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(a*B*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e -
b^3*e^2 - b*c*(c*d^2 - 3*a*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d
 - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*e
*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c
*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(a^(3/2)*B*Sqrt
[c]*e^2 + A*(c*d - b*e)^2 + a*e*(2*B*c*d - b*B*e - A*c*e) - Sqrt[a]*Sqrt[c]
*(B*c*d^2 - A*e*(2*c*d - b*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*(c*d^2 - b*
d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(B*d -
 A*e)*(3*c*d^2 - e*(2*b*d - a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4
*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*
e^2)^2*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*e*((Sqrt[c]*d)/Sqrt[a] + e)^2*(A
*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^
2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a
+ b*x^2 + c*x^4])
```

Rule 1720

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[p + 1/2] && IntegerQ[q]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; Ne
```

```
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A + Bx^2}{\left(d + ex^2\right)^2 \left(a + bx^2 + cx^4\right)^{3/2}} \, dx = \int \left(\frac{aBe(2cd - be) + A\left(c^2d^2 + b^2e^2 - ce(2bd + ae)\right) - c\left(Ae(2cd - be) - B\left(cd^2 - ae^2\right.\right.}{\left(cd^2 - bde + ae^2\right)^2 \left(a + bx^2 + cx^4\right)^{3/2}} \right.$$

$$= \frac{\int \frac{aBe(2cd-be) + A\left(c^2d^2 + b^2e^2 - ce(2bd+ae)\right) - c\left(Ae(2cd-be) - B\left(cd^2 - ae^2\right)\right)x^2}{\left(a+bx^2+cx^4\right)^{3/2}} \, dx}{\left(cd^2 - bde + ae^2\right)^2} - \frac{(e(Bd - Ae))\int \frac{}{\left(d+ex^2\right.}}{cd^2 - bde +}$$

$$= \frac{x\left(abc\left(Ae(2cd - be) - B\left(cd^2 - ae^2\right)\right) + \left(b^2 - 2ac\right)\left(aBe(2cd - be) + A\left(c^2d^2 + b^2e^2\right.\right.}{a\left(b^2 - 4ac\right)}$$

$$= \frac{x\left(abc\left(Ae(2cd - be) - B\left(cd^2 - ae^2\right)\right) + \left(b^2 - 2ac\right)\left(aBe(2cd - be) + A\left(c^2d^2 + b^2e^2\right.\right.}{a\left(b^2 - 4ac\right)}$$

$$= \frac{x\left(abc\left(Ae(2cd - be) - B\left(cd^2 - ae^2\right)\right) + \left(b^2 - 2ac\right)\left(aBe(2cd - be) + A\left(c^2d^2 + b^2e^2\right.\right.}{a\left(b^2 - 4ac\right)}$$

$$= \frac{x\left(abc\left(Ae(2cd - be) - B\left(cd^2 - ae^2\right)\right) + \left(b^2 - 2ac\right)\left(aBe(2cd - be) + A\left(c^2d^2 + b^2e^2\right.\right.}{a\left(b^2 - 4ac\right)}$$

Mathematica [C]   time = 8.00736, size = 8031, normalized size = 6.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [B]   time = 0.041, size = 8276, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^2), x)
```

3.30 $\int \dfrac{\sqrt{a}+\sqrt{c}x^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=273

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+c}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[Out] -((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*S
qrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a
*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4
*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A]   time = 0.135684, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {1706}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+c}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*S
qrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a
*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4
*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+}}$$

Mathematica [C]   time = 0.53715, size = 310, normalized size = 1.14

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\sqrt{c}d\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+\left(\sqrt{a}e-\sqrt{c}d\right)\Pi\left(\frac{\left(b+\sqrt{b^2-4a}\right)}{2cd}\right.\right.}{\sqrt{2}de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c]*d*EllipticF[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])] + (-(Sqrt[c]*d) + Sqrt[a]*e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]   time = 0.056, size = 365, normalized size = 1.3

$$\frac{\sqrt{2}}{4e}\sqrt{c}\sqrt{4-2\frac{\left(\sqrt{-4ac+b^2}-b\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(\sqrt{-4ac+b^2}-b\right)},\frac{1}{2}\sqrt{-4+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*c^(1/2)/e*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(
1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(e*a^(1/2)-d*c^(1/2))/e/d*2^(1/2)/(1/
a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*b*x
^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a
)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a
*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)/a)^(1/2))
```

Maxima [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a} + \sqrt{c}x^2}{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/2)+x**2*c**(1/2))/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((sqrt(a) + sqrt(c)*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)),
 x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c}x^2 + \sqrt{a}}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.31 $\quad \int \dfrac{1+\sqrt{\frac{c}{a}}x^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=271

$$\frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\Pi\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de};2\tan^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)|\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+bx^2+cx^4}}-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[Out] -((Sqrt[c/a]*d - e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]
*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
 + ((Sqrt[c/a]*d + e)*(1 + Sqrt[c/a]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 +
Sqrt[c/a]*x^2)^2)]*EllipticPi[-(Sqrt[c/a]*d - e)^2/(4*Sqrt[c/a]*d*e), 2*Arc
Tan[(c/a)^(1/4)*x], (2 - (b*Sqrt[c/a])/c)/4])/(4*(c/a)^(1/4)*d*e*Sqrt[a + b
*x^2 + c*x^4])
```

Rubi [A]   time = 0.186994, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {1706}

$$\frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\Pi\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de};2\tan^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)|\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+bx^2+cx^4}}-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((Sqrt[c/a]*d - e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]
*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
 + ((Sqrt[c/a]*d + e)*(1 + Sqrt[c/a]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 +
Sqrt[c/a]*x^2)^2)]*EllipticPi[-(Sqrt[c/a]*d - e)^2/(4*Sqrt[c/a]*d*e), 2*Arc
Tan[(c/a)^(1/4)*x], (2 - (b*Sqrt[c/a])/c)/4])/(4*(c/a)^(1/4)*d*e*Sqrt[a + b
*x^2 + c*x^4])
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{\frac{c}{a}}d-e\right)\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\left(\sqrt{\frac{c}{a}}d+e\right)\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(1+\sqrt{\frac{c}{a}}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{\frac{c}{a}}d-\right.}{4\sqrt{\frac{c}{a}}d}\right.}{4\sqrt[4]{\frac{c}{a}}de\sqrt{a+bx^2+}}$$

Mathematica [C]   time = 0.443384, size = 312, normalized size = 1.15

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(d\sqrt{\frac{c}{a}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+\left(e-d\sqrt{\frac{c}{a}}\right)\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\right.}{\sqrt{2}de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])] + (-(Sqrt[c/a]*d) + e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e
)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]   time = 0.042, size = 369, normalized size = 1.4

$$\frac{\sqrt{2}}{4e}\sqrt{\frac{c}{a}}\sqrt{4-2\frac{\left(\sqrt{-4ac+b^2}-b\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(\sqrt{-4ac+b^2}-b\right)},\frac{1}{2}\sqrt{-4+2\frac{b\left(\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^2*(1/a*c)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*(1/a*c)^(1/2)/e*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+
b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/e*(-d*(1/a*c)^(1/2)+e)/d*2^(1
/2)/(1/a*(-4*a*c+b^2)^(1/2)-1/a*b)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/
2/a*b*x^2)^(1/2)*(1+1/2/a*b*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+
b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2
/((-4*a*c+b^2)^(1/2)-b)*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

160

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [F(-1)]   time = 0., size = 0, normalized size = 0.

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{\frac{c}{a}} + 1}{\left(d + ex^2\right) \sqrt{a + bx^2 + cx^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**2*(c/a)**(1/2))/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((x**2*sqrt(c/a) + 1)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{\frac{c}{a}} + 1}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.32 $\int \dfrac{946+315x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=106

$$\frac{631\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{2525\left(x^2+2\right)\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt
[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2
])/(14*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4]))
```

Rubi [A] time = 0.105343, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.129, Rules used = {1704, 1099, 1456, 539}

$$\frac{631\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{2525\left(x^2+2\right)\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(946 + 315*x^2)/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt
[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2
])/(14*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4]))
```

Rule 1704

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Sqrt[b^2 - 4*a*c]}, Dist[(2*a*B - A
*(b + q))/(2*a*e - d*(b + q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[(B*d - A*e)/(2*a*e - d*(b + q)), Int[(2*a + (b + q)*x^2)/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x] /; RationalQ[q]] /; FreeQ[{a, b, c, d, e, A, B
}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*A^2
- b*A*B + a*B^2, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
```

```
0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{946 + 315x^2}{\left(7 + 5x^2\right)\sqrt{2 + 3x^2 + x^4}}\,dx = \frac{631}{2}\int \frac{1}{\sqrt{2 + 3x^2 + x^4}}\,dx - \frac{2525}{8}\int \frac{4 + 4x^2}{\left(7 + 5x^2\right)\sqrt{2 + 3x^2 + x^4}}\,dx$$

$$= \frac{631\left(1 + x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2 + 3x^2 + x^4}} - \frac{\left(2525\sqrt{\frac{1}{2} + \frac{x^2}{4}}\sqrt{4 + 4x^2}\right)\int \frac{\sqrt{4+4x^2}}{\sqrt{\frac{1}{2} + \frac{x^2}{4}}\left(7+5x^2\right)}\,dx}{8\sqrt{2 + 3x^2 + x^4}}$$

$$= \frac{631\left(1 + x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2 + 3x^2 + x^4}} - \frac{2525\left(2 + x^2\right)\Pi\left(\frac{2}{7}; \tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2 + 3x^2 + x^4}}$$

Mathematica [C]   time = 0.182633, size = 74, normalized size = 0.7

$$-\frac{i\sqrt{x^2 + 1}\sqrt{x^2 + 2}\left(441\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right) + 505\Pi\left(\frac{10}{7}; i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|2\right)\right)}{7\sqrt{x^4 + 3x^2 + 2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(946 + 315*x^2)/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I/7)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(441*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
 + 505*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C]   time = 0.028, size = 93, normalized size = 0.9

$$-\frac{63\,i}{2}\sqrt{2}\,\text{EllipticF}\left(\frac{i}{2}x\sqrt{2}, \sqrt{2}\right)\sqrt{2x^2 + 4}\sqrt{x^2 + 1}\frac{1}{\sqrt{x^4 + 3x^2 + 2}} - \frac{505\,i}{7}\sqrt{2}\sqrt{1 + \frac{x^2}{2}}\sqrt{x^2 + 1}\,\text{EllipticPi}\left(\frac{i}{2}x\sqrt{2}, \frac{10}{7}, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -63/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*x*2^(1/2),2^(1/2))-505/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{315\,x^2 + 946}{\sqrt{x^4 + 3\,x^2 + 2}\left(5\,x^2 + 7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((315*x^2 + 946)/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 2}\left(315x^2 + 946\right)}{5x^6 + 22x^4 + 31x^2 + 14}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(315*x^2 + 946)/(5*x^6 + 22*x^4 + 31*x^2 + 1
4), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{315x^2 + 946}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}\left(5x^2 + 7\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((315*x**2+946)/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((315*x**2 + 946)/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{315x^2 + 946}{\sqrt{x^4 + 3x^2 + 2}\left(5x^2 + 7\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((315*x^2 + 946)/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

3.33 $\quad \int \dfrac{\left(A+Bx^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=218

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b-\sqrt{b^2-4ac}}+\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)F_1\left(\frac{1}{2};1\right.}{\sqrt{b^2-4ac}+b}$$

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q,
 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b - Sqrt[b^2 - 4
*a*c])*(1 + (e*x^2)/d)^q) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e
*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*
x^2)/d)])/((b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)
```

Rubi [A]   time = 0.463449, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1692, 430, 429}

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b-\sqrt{b^2-4ac}}+\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)F_1\left(\frac{1}{2};1\right.}{\sqrt{b^2-4ac}+b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q,
 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b - Sqrt[b^2 - 4
*a*c])*(1 + (e*x^2)/d)^q) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e
*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*
x^2)/d)])/((b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^q}{a + bx^2 + cx^4}\,dx = \int \left(\frac{\left(B + \frac{-bB + 2Ac}{\sqrt{b^2 - 4ac}}\right)\left(d + ex^2\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2} + \frac{\left(B - \frac{-bB + 2Ac}{\sqrt{b^2 - 4ac}}\right)\left(d + ex^2\right)^q}{b + \sqrt{b^2 - 4ac} + 2cx^2}\right)dx$$

$$= \left(B - \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right)\int \frac{\left(d + ex^2\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2}\,dx + \left(B + \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right)\int \frac{\left(d + ex^2\right)^q}{b + \sqrt{b^2 - 4ac} + 2cx^2}$$

$$= \left(\left(B - \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right)\left(d + ex^2\right)^q\left(1 + \frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1 + \frac{ex^2}{d}\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2}\,dx + \left(\left(B + \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right)\right.$$

$$= \frac{\left(B - \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right)x\left(d + ex^2\right)^q\left(1 + \frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2}; 1, -q; \frac{3}{2}; -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{ex^2}{d}\right)}{b - \sqrt{b^2 - 4ac}} + \frac{\left(B + \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right)x\left(d\right.}{}$$

Mathematica [F]   time = 0.241547, size = 0, normalized size = 0.

$$\int \frac{\left(A + Bx^2\right)\left(d + ex^2\right)^q}{a + bx^2 + cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]   time = 0.042, size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)\left(ex^2 + d\right)^q}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

3.34 $\int \dfrac{x\left(1+2x^2\right)}{\sqrt{1+x^2}\left(1+x^2+x^4\right)}\, dx$

Optimal. Leaf size=106

$$\frac{1}{4}\sqrt{3}\log\left(x^2-\sqrt{3}\sqrt{x^2+1}+2\right)-\frac{1}{4}\sqrt{3}\log\left(x^2+\sqrt{3}\sqrt{x^2+1}+2\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt{x^2+1}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt{x^2+1}+\sqrt{3}\right.$$

```
[Out] -ArcTan[Sqrt[3] - 2*Sqrt[1 + x^2]]/2 + ArcTan[Sqrt[3] + 2*Sqrt[1 + x^2]]/2
+ (Sqrt[3]*Log[2 + x^2 - Sqrt[3]*Sqrt[1 + x^2]])/4 - (Sqrt[3]*Log[2 + x^2 +
 Sqrt[3]*Sqrt[1 + x^2]])/4
```

Rubi [A]   time = 0.128522, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1685, 826, 1169, 634, 618, 204, 628}

$$\frac{1}{4}\sqrt{3}\log\left(x^2-\sqrt{3}\sqrt{x^2+1}+2\right)-\frac{1}{4}\sqrt{3}\log\left(x^2+\sqrt{3}\sqrt{x^2+1}+2\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt{x^2+1}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt{x^2+1}+\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + 2*x^2))/(Sqrt[1 + x^2]*(1 + x^2 + x^4)),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*Sqrt[1 + x^2]]/2 + ArcTan[Sqrt[3] + 2*Sqrt[1 + x^2]]/2
+ (Sqrt[3]*Log[2 + x^2 - Sqrt[3]*Sqrt[1 + x^2]])/4 - (Sqrt[3]*Log[2 + x^2 +
 Sqrt[3]*Sqrt[1 + x^2]])/4
```

Rule 1685

```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)
^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
&& PolyQ[Px, x^2]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(1 + 2x^2\right)}{\sqrt{1 + x^2}\left(1 + x^2 + x^4\right)}\, dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1 + 2x}{\sqrt{1 + x}\left(1 + x + x^2\right)}\, dx, x, x^2\right)$$

$$= \text{Subst}\left(\int \frac{-1 + 2x^2}{1 - x^2 + x^4}\, dx, x, \sqrt{1 + x^2}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{-\sqrt{3} + 3x}{1 - \sqrt{3}x + x^2}\, dx, x, \sqrt{1 + x^2}\right)}{2\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt{3} - 3x}{1 + \sqrt{3}x + x^2}\, dx, x, \sqrt{1 + x^2}\right)}{2\sqrt{3}}$$

$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1 - \sqrt{3}x + x^2}\, dx, x, \sqrt{1 + x^2}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1 + \sqrt{3}x + x^2}\, dx, x, \sqrt{1 + x^2}\right)$$

$$= \frac{1}{4}\sqrt{3}\log\left(2 + x^2 - \sqrt{3}\sqrt{1 + x^2}\right) - \frac{1}{4}\sqrt{3}\log\left(2 + x^2 + \sqrt{3}\sqrt{1 + x^2}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1 - }\right.$$

$$= -\frac{1}{2}\tan^{-1}\left(\sqrt{3} - 2\sqrt{1 + x^2}\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3} + 2\sqrt{1 + x^2}\right) + \frac{1}{4}\sqrt{3}\log\left(2 + x^2 - \sqrt{3}\sqrt{1 + }\right.$$

Mathematica [C]   time = 0.201181, size = 125, normalized size = 1.18

$$\frac{\sqrt{2\left(1 - i\sqrt{3}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^2+1}}{\sqrt{1 - i\sqrt{3}}}\right)}{-1 + i\sqrt{3}} + \frac{\sqrt{2\left(1 + i\sqrt{3}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^2+1}}{\sqrt{1 + i\sqrt{3}}}\right)}{-1 - i\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + 2*x^2))/(Sqrt[1 + x^2]*(1 + x^2 + x^4)),x]
```

```
[Out] (Sqrt[2*(1 - I*Sqrt[3])]*ArcTanh[(Sqrt[2]*Sqrt[1 + x^2])/Sqrt[1 - I*Sqrt[3]
]])/(-1 + I*Sqrt[3]) + (Sqrt[2*(1 + I*Sqrt[3])]*ArcTanh[(Sqrt[2]*Sqrt[1 + x
^2])/Sqrt[1 + I*Sqrt[3]]])/(-1 - I*Sqrt[3])
```

Maple [B]   time = 0.053, size = 296, normalized size = 2.8

$$-\frac{\sqrt{2}}{4}\sqrt{2\frac{(1 + x)^2}{(1 - x)^2} + 2}\left(\sqrt{3}\text{Artanh}\left(\frac{\sqrt{3}}{2}\sqrt{2\frac{(1 + x)^2}{(1 - x)^2} + 2}\right) + \arctan\left(\frac{1 + x}{1 - x}\sqrt{2\frac{(1 + x)^2}{(1 - x)^2} + 2}\left(\frac{(1 + x)^2}{(1 - x)^2} + 1\right)^{-1}\right)\right)\frac{1}{\sqrt{\left(\frac{(1 + x)}{(1 - x)}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(1+x)^2/(1
-x)^2+2)^(1/2)*3^(1/2))+arctan(1/((1+x)^2/(1-x)^2+1)*(2*(1+x)^2/(1-x)^2+2)^
(1/2)*(1+x)/(1-x)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1
-x)+1)-1/4*2^(1/2)*(2*(x-1)^2/(-1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(x-
1)^2/(-1-x)^2+2)^(1/2)*3^(1/2))+arctan(1/((x-1)^2/(-1-x)^2+1)*(2*(x-1)^2/(-
1-x)^2+2)^(1/2)*(x-1)/(-1-x)))/(((x-1)^2/(-1-x)^2+1)/((x-1)/(-1-x)+1)^2)^(1
/2)/((x-1)/(-1-x)+1)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2+1)x}{(x^4+x^2+1)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)*x/((x^4 + x^2 + 1)*sqrt(x^2 + 1)), x)
```

Fricas [B]   time = 2.41911, size = 744, normalized size = 7.02

$$\frac{1}{4}\sqrt{3}\log\left(32x^4+80x^2+32\sqrt{3}(x^3+x)-16\left(2x^3+\sqrt{3}(2x^2+1)+4x\right)\sqrt{x^2+1}+32\right)-\frac{1}{4}\sqrt{3}\log\left(32x^4+80x^2-32\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*log(32*x^4 + 80*x^2 + 32*sqrt(3)*(x^3 + x) - 16*(2*x^3 + sqrt(3
)*(2*x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 32) - 1/4*sqrt(3)*log(32*x^4 + 80*x^2
- 32*sqrt(3)*(x^3 + x) - 16*(2*x^3 - sqrt(3)*(2*x^2 + 1) + 4*x)*sqrt(x^2 +
1) + 32) - arctan(2*sqrt(2*x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) - (2*x^3 + sqr
t(3)*(2*x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 2)*(x + sqrt(x^2 + 1)) + sqrt(3) -
2*sqrt(x^2 + 1)) - arctan(2*sqrt(2*x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) - (2*x
^3 - sqrt(3)*(2*x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 2)*(x + sqrt(x^2 + 1)) - sq
rt(3) - 2*sqrt(x^2 + 1))
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{x(2x^2+1)}{\sqrt{x^2+1}(x^2-x+1)(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2+1)/(x**4+x**2+1)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(x*(2*x**2 + 1)/(sqrt(x**2 + 1)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 + 1\right)x}{\left(x^4 + x^2 + 1\right)\sqrt{x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)*x/((x^4 + x^2 + 1)*sqrt(x^2 + 1)), x)
```

3.35 $\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx$

Optimal. Leaf size=145

$$\frac{\sqrt{2\sqrt{a}\sqrt{c}+b}\,\tanh^{-1}\left(\frac{x\sqrt{2\sqrt{a}\sqrt{c}+b}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{cd}} - \frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\,\tanh^{-1}\left(\frac{x\sqrt{b-2\sqrt{a}\sqrt{c}}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{cd}}$$

```
[Out] -(Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*ArcTanh[(Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*x)/Sqrt[
a + b*x^2 + c*x^4]])/(4*Sqrt[a]*Sqrt[c]*d) + (Sqrt[b + 2*Sqrt[a]*Sqrt[c]]*A
rcTanh[(Sqrt[b + 2*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^2 + c*x^4]])/(4*Sqrt[a]
*Sqrt[c]*d)
```

Rubi [A]   time = 0.207419, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2071, 1093, 208}

$$\frac{\sqrt{2\sqrt{a}\sqrt{c}+b}\,\tanh^{-1}\left(\frac{x\sqrt{2\sqrt{a}\sqrt{c}+b}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{cd}} - \frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\,\tanh^{-1}\left(\frac{x\sqrt{b-2\sqrt{a}\sqrt{c}}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{cd}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(a*d - c*d*x^4),x]
```

```
[Out] -(Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*ArcTanh[(Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*x)/Sqrt[
a + b*x^2 + c*x^4]])/(4*Sqrt[a]*Sqrt[c]*d) + (Sqrt[b + 2*Sqrt[a]*Sqrt[c]]*A
rcTanh[(Sqrt[b + 2*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^2 + c*x^4]])/(4*Sqrt[a]
*Sqrt[c]*d)
```

Rule 2071

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-2bx^2+\left(b^2-4ac\right)x^4}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)}{d}$$

$$= \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{-b-2\sqrt{a}\sqrt{c}+\left(b^2-4ac\right)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} - \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{-b+2\sqrt{a}\sqrt{c}+\left(b^2\right.}\right.}{4\sqrt{a}\sqrt{c}d}$$

$$= -\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} + \frac{\sqrt{b+2\sqrt{a}\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{b+2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d}$$

Mathematica [C]   time = 0.661426, size = 441, normalized size = 3.04

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(2\sqrt{a}\sqrt{c}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right), \frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right) - \left(2\sqrt{a}\sqrt{c}+b\right)\Pi\left(\frac{-b-\sqrt{b^2-}}{2\sqrt{a}\sqrt{}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d\sqrt{\frac{c}{\sqrt{b^2-4ac+b}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(a*d - c*d*x^4),x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(2*Sqrt[a]*Sqrt[c]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])] - (b + 2*Sqrt[a]*Sqrt[c])*EllipticPi[(-b - Sqrt[b^2 -
 4*a*c])/(2*Sqrt[a]*Sqrt[c]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b - 2*Sqrt[a]*
Sqrt[c])*EllipticPi[(b + Sqrt[b^2 - 4*a*c])/(2*Sqrt[a]*Sqrt[c]), I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*d*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B]   time = 0.033, size = 238, normalized size = 1.6

$$-\frac{\sqrt{2}}{2d}\arctan\left(\frac{\sqrt{2}}{x}\sqrt{cx^4+bx^2+a}\,\frac{1}{\sqrt{4\sqrt{ac}-2b}}\right)\frac{1}{\sqrt{4\sqrt{ac}-2b}} + \frac{\sqrt{2}b}{4d}\arctan\left(\frac{\sqrt{2}}{x}\sqrt{cx^4+bx^2+a}\,\frac{1}{\sqrt{4\sqrt{ac}-2b}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x)
```

```
[Out] -1/2/d*2^(1/2)/(4*(a*c)^(1/2)-2*b)^(1/2)*arctan((c*x^4+b*x^2+a)^(1/2)*2^(1/
2)/x/(4*(a*c)^(1/2)-2*b)^(1/2))+1/4/d*2^(1/2)/(a*c)^(1/2)/(4*(a*c)^(1/2)-2*
b)^(1/2)*arctan((c*x^4+b*x^2+a)^(1/2)*2^(1/2)/x/(4*(a*c)^(1/2)-2*b)^(1/2))*
b-1/2/d*2^(1/2)/(-4*(a*c)^(1/2)-2*b)^(1/2)*arctan((c*x^4+b*x^2+a)^(1/2)*2^(
1/2)/x/(-4*(a*c)^(1/2)-2*b)^(1/2))-1/4/d*2^(1/2)/(a*c)^(1/2)/(-4*(a*c)^(1/2
)-2*b)^(1/2)*arctan((c*x^4+b*x^2+a)^(1/2)*2^(1/2)/x/(-4*(a*c)^(1/2)-2*b)^(1
/2))*b
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{cx^4 + bx^2 + a}}{cdx^4 - ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^4 + b*x^2 + a)/(c*d*x^4 - a*d), x)
```

Fricas [B]   time = 22.774, size = 1211, normalized size = 8.35

$$\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}+x^2\right)+\left(acd^3x^3\sqrt{\frac{1}{acd^4}}+adx\right)\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}}{cx^4-a}\right) - \frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+}{acd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(c*x^4 + b*x
^2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) + x^2) + (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) +
a*d*x)*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4 - a)) - 1/
8*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(c*x^4 + b*x^2
 + a)*(a*d^2*sqrt(1/(a*c*d^4)) + x^2) - (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) + a*
d*x)*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4 - a)) + 1/8*
sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(c*x^4 + b*x^2
 + a)*(a*d^2*sqrt(1/(a*c*d^4)) - x^2) + (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) - a*
d*x)*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 - a)) - 1/8
*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(c*x^4 + b*x^
2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) - x^2) - (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) - a
*d*x)*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 - a))
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{a+bx^2+cx^4}}{-a+cx^4}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(-c*d*x**4+a*d),x)
```

```
[Out] -Integral(sqrt(a + b*x**2 + c*x**4)/(-a + c*x**4), x)/d
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{cx^4 + bx^2 + a}}{cdx^4 - ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(c*x^4 + b*x^2 + a)/(c*d*x^4 - a*d), x)
```

3.36 $\int \dfrac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx$

Optimal. Leaf size=239

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\,\tanh^{-1}\left(\frac{x\sqrt{\sqrt{4ac+b^2}-b}\left(\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{cd}} - \frac{\sqrt{\sqrt{4ac+b^2}+b}\,\tan^{-1}\left(\frac{x\sqrt{\sqrt{4ac+b^2}+b}\left(-\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{cd}}$$

```
[Out] -(Sqrt[b + Sqrt[b^2 + 4*a*c]]*ArcTan[(Sqrt[b + Sqrt[b^2 + 4*a*c]]*x*(b - Sq
rt[b^2 + 4*a*c] - 2*c*x^2))/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2 - c*x
^4])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*d) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*ArcTan
h[(Sqrt[-b + Sqrt[b^2 + 4*a*c]]*x*(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2))/(2*Sqr
t[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*
d)
```

Rubi [A]   time = 0.184369, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.033, Rules used = {2072}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\,\tanh^{-1}\left(\frac{x\sqrt{\sqrt{4ac+b^2}-b}\left(\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{cd}} - \frac{\sqrt{\sqrt{4ac+b^2}+b}\,\tan^{-1}\left(\frac{x\sqrt{\sqrt{4ac+b^2}+b}\left(-\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{cd}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 - c*x^4]/(a*d + c*d*x^4),x]
```

```
[Out] -(Sqrt[b + Sqrt[b^2 + 4*a*c]]*ArcTan[(Sqrt[b + Sqrt[b^2 + 4*a*c]]*x*(b - Sq
rt[b^2 + 4*a*c] - 2*c*x^2))/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2 - c*x
^4])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*d) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*ArcTan
h[(Sqrt[-b + Sqrt[b^2 + 4*a*c]]*x*(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2))/(2*Sqr
t[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*
d)
```

Rule 2072

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, -Simp[(a*Sqrt[b + q]*ArcTan[(Sqrt[b + q
]*x*(b - q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x] + Simp[(a*Sqrt[-b + q]*ArcTanh[(Sqrt[-b + q
]*x*(b + q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d +
a*e, 0] && NegQ[a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = -\frac{\sqrt{b+\sqrt{b^2+4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+\sqrt{b^2+4ac}}\,x\left(b-\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{cd}} + \frac{\sqrt{-b+\sqrt{b^2+4ac}}\,\tanh^{-1}\left(\frac{\sqrt{-b+\sqrt{b^2+4ac}}}{2\sqrt{2}\sqrt{a}\cdots}\right)}{2\sqrt{2}\sqrt{a}\sqrt{cd}}$$

Mathematica [C]   time = 0.693984, size = 432, normalized size = 1.81

$$\sqrt{\frac{4cx^2}{\sqrt{4ac+b^2}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(2i\sqrt{a}\sqrt{c}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)+\left(b-2i\sqrt{a}\sqrt{c}\right)\Pi\left(-\frac{i\left(b+\right.}{2}\right.\right.}{4\sqrt{a}\sqrt{c}d\sqrt{-\frac{}{\sqrt{4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 - c*x^4]/(a*d + c*d*x^4),x]
```

```
[Out] (Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*((2*I)*Sqrt[a]*Sqrt[c]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/
(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c
])] + (b - (2*I)*Sqrt[a]*Sqrt[c])*EllipticPi[((-I/2)*(b + Sqrt[b^2 + 4*a*c]
))/(Sqrt[a]*Sqrt[c]), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*
x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] - (b + (2*I)*Sqrt[a]*S
qrt[c])*EllipticPi[((I/2)*(b + Sqrt[b^2 + 4*a*c]))/(Sqrt[a]*Sqrt[c]), I*Arc
Sinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])
/(b - Sqrt[b^2 + 4*a*c])]))/(4*Sqrt[a]*Sqrt[c]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a
*c]))]*d*Sqrt[a + b*x^2 - c*x^4])
```

Maple [B]   time = 0.072, size = 568, normalized size = 2.4

$$-\frac{\sqrt{2}b}{32\,acd}\sqrt{b+\sqrt{4\,ac+b^2}}\ln\left(\frac{-cx^4+bx^2+a}{x^2}+\frac{\sqrt{2}}{x}\sqrt{-cx^4+bx^2+a}\sqrt{b+\sqrt{4\,ac+b^2}}+\sqrt{4\,ac+b^2}\right)+\frac{\sqrt{2}}{32\,acd}\sqrt{b+\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x)
```

```
[Out] -1/32/d*2^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)/a/c*b*ln((-c*x^4+b*x^2+a)/x^2+(
-c*x^4+b*x^2+a)^(1/2)*2^(1/2)/x*(b+(4*a*c+b^2)^(1/2))^(1/2)+(4*a*c+b^2)^(1/
2))+1/32/d*2^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)/a/c*(4*a*c+b^2)^(1/2)*ln((-c
*x^4+b*x^2+a)/x^2+(-c*x^4+b*x^2+a)^(1/2)*2^(1/2)/x*(b+(4*a*c+b^2)^(1/2))^(1
/2)+(4*a*c+b^2)^(1/2))-1/4/d*2^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2)*arctan(1/
2*(2*(-c*x^4+b*x^2+a)^(1/2)*2^(1/2)/x+2*(b+(4*a*c+b^2)^(1/2))^(1/2))/(-b+(4
*a*c+b^2)^(1/2))^(1/2))+1/32/d*2^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)/a/c*b*ln
((-c*x^4+b*x^2+a)^(1/2)*2^(1/2)/x*(b+(4*a*c+b^2)^(1/2))^(1/2)-(-c*x^4+b*x^2
+a)/x^2-(4*a*c+b^2)^(1/2))-1/32/d*2^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)/a/c*(
4*a*c+b^2)^(1/2)*ln((-c*x^4+b*x^2+a)^(1/2)*2^(1/2)/x*(b+(4*a*c+b^2)^(1/2))^
(1/2)-(-c*x^4+b*x^2+a)/x^2-(4*a*c+b^2)^(1/2))+1/4/d*2^(1/2)/(-b+(4*a*c+b^2)
^(1/2))^(1/2)*arctan(1/2*(2*(b+(4*a*c+b^2)^(1/2))^(1/2)-2*(-c*x^4+b*x^2+a)^
(1/2)*2^(1/2)/x)/(-b+(4*a*c+b^2)^(1/2))^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-cx^4+bx^2+a}}{cdx^4+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c*x^4 + b*x^2 + a)/(c*d*x^4 + a*d), x)
```

Fricas [B]   time = 22.6723, size = 1364, normalized size = 5.71

$$-\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}-b}{acd^2}}\,\log\left(-\frac{\sqrt{-cx^4+bx^2+a}\,ad^2\sqrt{-\frac{1}{acd^4}}+\sqrt{-cx^4+bx^2+a}\,ax^2+\left(acd^3x^3\sqrt{-\frac{1}{acd^4}}-adx\right)\sqrt{\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}-}}{acd^2}}}{cx^4+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(-c*x^4 +
 b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) + sqrt(-c*x^4 + b*x^2 + a)*x^2 + (a*c*
d^3*x^3*sqrt(-1/(a*c*d^4)) - a*d*x)*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)
/(a*c*d^2)))/(c*x^4 + a)) + 1/8*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)/(a*
c*d^2))*log(-(sqrt(-c*x^4 + b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) + sqrt(-c*x
^4 + b*x^2 + a)*x^2 - (a*c*d^3*x^3*sqrt(-1/(a*c*d^4)) - a*d*x)*sqrt((2*a*c*
d^2*sqrt(-1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 + a)) - 1/8*sqrt(-(2*a*c*d^2
*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(-c*x^4 + b*x^2 + a)*a*d^2*sqr
t(-1/(a*c*d^4)) - sqrt(-c*x^4 + b*x^2 + a)*x^2 + (a*c*d^3*x^3*sqrt(-1/(a*c*
d^4)) + a*d*x)*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4
+ a)) + 1/8*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(-
c*x^4 + b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) - sqrt(-c*x^4 + b*x^2 + a)*x^2
- (a*c*d^3*x^3*sqrt(-1/(a*c*d^4)) + a*d*x)*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^
4)) + b)/(a*c*d^2)))/(c*x^4 + a))
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+bx^2-cx^4}}{a+cx^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+b*x**2+a)**(1/2)/(c*d*x**4+a*d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 - c*x**4)/(a + c*x**4), x)/d
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-cx^4+bx^2+a}}{cdx^4+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c*x^4 + b*x^2 + a)/(c*d*x^4 + a*d), x)
```

3.37 $\quad \int x\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\left(c + dx^2 + ex\right)^{3/2}\left(-80ad^2 + 32bcd + 42bdex - 35be^2\right)}{240d^3\left(a + bx^2\right)} + \frac{e\sqrt{a^2 + 2abx^2 + b^2x^4}(2dx + e)\sqrt{c + dx^2}}{128d^4\left(a + b\right)}$$

```
[Out] (e*(12*b*c*d - 16*a*d^2 - 7*b*e^2)*(e + 2*d*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(128*d^4*(a + b*x^2)) + (b*x^2*(c + e*x + d*x^2)
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) - ((32*b*c*d - 80
*a*d^2 - 35*b*e^2 + 42*b*d*e*x)*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(240*d^3*(a + b*x^2)) + (e*(4*c*d - e^2)*(12*b*c*d - 16*a*d^2
 - 7*b*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*
Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x^2))
```

Rubi [A]   time = 0.625224, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6744, 1653, 779, 612, 621, 206}

$$-\frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\left(c + dx^2 + ex\right)^{3/2}\left(-80ad^2 + 32bcd + 42bdex - 35be^2\right)}{240d^3\left(a + bx^2\right)} + \frac{e\sqrt{a^2 + 2abx^2 + b^2x^4}(2dx + e)\sqrt{c + dx^2}}{128d^4\left(a + b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*(12*b*c*d - 16*a*d^2 - 7*b*e^2)*(e + 2*d*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(128*d^4*(a + b*x^2)) + (b*x^2*(c + e*x + d*x^2)
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) - ((32*b*c*d - 80
*a*d^2 - 35*b*e^2 + 42*b*d*e*x)*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(240*d^3*(a + b*x^2)) + (e*(4*c*d - e^2)*(12*b*c*d - 16*a*d^2
 - 7*b*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*
Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x^2))
```

Rule 6744

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\int x\left(2ab + 2b^2x^2\right)\sqrt{c + ex + dx^2}\,dx}{2ab + 2b^2x^2}$$

$$= \frac{bx^2\left(c + ex + dx^2\right)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{5d\left(a + bx^2\right)} + \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\int x\left(-2b(2b\right.}{5d\left(2ab\right.}$$

$$= \frac{bx^2\left(c + ex + dx^2\right)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{5d\left(a + bx^2\right)} - \frac{\left(32bcd - 80ad^2 - 35be^2 + 42bde\right.}{240a}$$

$$= \frac{e\left(12bcd - 16ad^2 - 7be^2\right)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{128d^4\left(a + bx^2\right)} + \frac{bx^2\left(\right.}{}$$

$$= \frac{e\left(12bcd - 16ad^2 - 7be^2\right)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{128d^4\left(a + bx^2\right)} + \frac{bx^2\left(\right.}{}$$

$$= \frac{e\left(12bcd - 16ad^2 - 7be^2\right)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{128d^4\left(a + bx^2\right)} + \frac{bx^2\left(\right.}{}$$

Mathematica [A]   time = 0.345258, size = 214, normalized size = 0.69

$$\frac{\sqrt{\left(a + bx^2\right)^2}\left(2\sqrt{d}\sqrt{c + x(dx + e)}\left(80ad^2\left(8cd + 8d^2x^2 + 2dex - 3e^2\right) + b\left(-256c^2d^2 + 4cd\left(32d^2x^2 - 58dex + 115e^2\right) - 56\right.\right.\right.}{3840d^{9/2}\left(a + \right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(80*a*d^2*(8*c*d - 3*
e^2 + 2*d*e*x + 8*d^2*x^2) + b*(-256*c^2*d^2 - 105*e^4 + 70*d*e^3*x - 56*d^
2*e^2*x^2 + 48*d^3*e*x^3 + 384*d^4*x^4 + 4*c*d*(115*e^2 - 58*d*e*x + 32*d^2
*x^2))) + 15*e*(-4*c*d + e^2)*(-12*b*c*d + 16*a*d^2 + 7*b*e^2)*ArcTanh[(e +
 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]]))/(3840*d^(9/2)*(a + b*x^2))
```

Maple [A]   time = 0.018, size = 442, normalized size = 1.4

$$\frac{1}{3840\,bx^2 + 3840\,a}\sqrt{\left(bx^2 + a\right)^2}\left(768\,d^{9/2}\left(x^2d + ex + c\right)^{3/2}x^2b - 672\,d^{7/2}\left(x^2d + ex + c\right)^{3/2}xbe + 1280\,d^{9/2}\left(x^2d + ex + \cdots\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3840*((b*x^2+a)^2)^(1/2)*(768*d^(9/2)*(d*x^2+e*x+c)^(3/2)*x^2*b-672*d^(7/
2)*(d*x^2+e*x+c)^(3/2)*x*b*e+1280*d^(9/2)*(d*x^2+e*x+c)^(3/2)*a-512*d^(7/2)
*(d*x^2+e*x+c)^(3/2)*b*c+560*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b*e^2-960*d^(9/2)*
(d*x^2+e*x+c)^(1/2)*x*a*e+720*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*b*c*e-420*d^(5/
2)*(d*x^2+e*x+c)^(1/2)*x*b*e^3-480*d^(7/2)*(d*x^2+e*x+c)^(1/2)*a*e^2+360*d^
(5/2)*(d*x^2+e*x+c)^(1/2)*b*c*e^2-210*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^4-960
*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*a*c*d^4*e+240*ln(1
/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*a*d^3*e^3+720*ln(1/2*(2
*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*c^2*d^3*e-600*ln(1/2*(2*(d
*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*c*d^2*e^3+105*ln(1/2*(2*(d*x^
2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e^5)/(b*x^2+a)/d^(11/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + ex + c}\sqrt{\left(bx^2 + a\right)^2}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x^2 + a)^2)*x, x)
```

Fricas [A]   time = 2.05082, size = 1118, normalized size = 3.62

$$\left[\frac{15\left(7\,be^5 - 8\left(5\,bcd - 2\,ad^2\right)e^3 + 16\left(3\,bc^2d^2 - 4\,acd^3\right)e\right)\sqrt{d}\log\left(8\,d^2x^2 + 8\,dex + 4\sqrt{dx^2 + ex + c}\left(2\,dx + e\right)\sqrt{d} + 4\,cd + \cdots\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(7*b*e^5 - 8*(5*b*c*d - 2*a*d^2)*e^3 + 16*(3*b*c^2*d^2 - 4*a*c*
d^3)*e)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(d) + 4*c*d + e^2) + 4*(384*b*d^5*x^4 + 48*b*d^4*e*x^3 - 256*b*c^2*d
^3 + 640*a*c*d^4 - 105*b*d*e^4 + 20*(23*b*c*d^2 - 12*a*d^3)*e^2 + 8*(16*b*c
*d^4 + 80*a*d^5 - 7*b*d^3*e^2)*x^2 + 2*(35*b*d^2*e^3 - 4*(29*b*c*d^3 - 20*a
*d^4)*e)*x)*sqrt(d*x^2 + e*x + c))/d^5, -1/3840*(15*(7*b*e^5 - 8*(5*b*c*d -
 2*a*d^2)*e^3 + 16*(3*b*c^2*d^2 - 4*a*c*d^3)*e)*sqrt(-d)*arctan(1/2*sqrt(d*
x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - 2*(384*b*d^5
*x^4 + 48*b*d^4*e*x^3 - 256*b*c^2*d^3 + 640*a*c*d^4 - 105*b*d*e^4 + 20*(23*
b*c*d^2 - 12*a*d^3)*e^2 + 8*(16*b*c*d^4 + 80*a*d^5 - 7*b*d^3*e^2)*x^2 + 2*(
35*b*d^2*e^3 - 4*(29*b*c*d^3 - 20*a*d^4)*e)*x)*sqrt(d*x^2 + e*x + c))/d^5]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]   time = 1.1943, size = 486, normalized size = 1.57

$$\frac{1}{1920}\sqrt{dx^2 + xe + c}\left(2\left(4\left(6\left(8\,bx\mathrm{sgn}\left(bx^2 + a\right) + \frac{be\mathrm{sgn}\left(bx^2 + a\right)}{d}\right)x + \frac{16\,bcd^3\mathrm{sgn}\left(bx^2 + a\right) + 80\,ad^4\mathrm{sgn}\left(bx^2 + a\right) - 7\,ba}{d^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(d*x^2 + x*e + c)*(2*(4*(6*(8*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^2
 + a)/d)*x + (16*b*c*d^3*sgn(b*x^2 + a) + 80*a*d^4*sgn(b*x^2 + a) - 7*b*d^2
*e^2*sgn(b*x^2 + a))/d^4)*x - (116*b*c*d^2*e*sgn(b*x^2 + a) - 80*a*d^3*e*sg
n(b*x^2 + a) - 35*b*d*e^3*sgn(b*x^2 + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x^2
 + a) - 640*a*c*d^3*sgn(b*x^2 + a) - 460*b*c*d*e^2*sgn(b*x^2 + a) + 240*a*d
^2*e^2*sgn(b*x^2 + a) + 105*b*e^4*sgn(b*x^2 + a))/d^4) - 1/256*(48*b*c^2*d^
2*e*sgn(b*x^2 + a) - 64*a*c*d^3*e*sgn(b*x^2 + a) - 40*b*c*d*e^3*sgn(b*x^2 +
 a) + 16*a*d^2*e^3*sgn(b*x^2 + a) + 7*b*e^5*sgn(b*x^2 + a))*log(abs(-2*(sqr
t(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(9/2)
```

3.38 $\quad \int \sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}\,dx$

Optimal. Leaf size=283

$$-\frac{\sqrt{a^2 + 2abx^2 + b^2x^4}(2dx + e)\sqrt{c + dx^2 + ex}\left(-16ad^2 + 4bcd - 5be^2\right)}{64d^3\left(a + bx^2\right)} - \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\left(4cd - e^2\right)\left(-16ad^2 + 4bc\right.}{128d^{7/2}\left(a + bx\right.}$$

```
[Out] -((4*b*c*d - 16*a*d^2 - 5*b*e^2)*(e + 2*d*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(64*d^3*(a + b*x^2)) - (5*b*e*(c + e*x + d*x^2)^(3
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*d^2*(a + b*x^2)) + (b*x*(c + e*x +
 d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a + b*x^2)) - ((4*c*d
 - e^2)*(4*b*c*d - 16*a*d^2 - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTa
nh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x^2)
)
```

Rubi [A]   time = 0.340535, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {6744, 1661, 640, 612, 621, 206}

$$-\frac{\sqrt{a^2 + 2abx^2 + b^2x^4}(2dx + e)\sqrt{c + dx^2 + ex}\left(-16ad^2 + 4bcd - 5be^2\right)}{64d^3\left(a + bx^2\right)} - \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\left(4cd - e^2\right)\left(-16ad^2 + 4bc\right.}{128d^{7/2}\left(a + bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((4*b*c*d - 16*a*d^2 - 5*b*e^2)*(e + 2*d*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(64*d^3*(a + b*x^2)) - (5*b*e*(c + e*x + d*x^2)^(3
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*d^2*(a + b*x^2)) + (b*x*(c + e*x +
 d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a + b*x^2)) - ((4*c*d
 - e^2)*(4*b*c*d - 16*a*d^2 - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTa
nh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x^2)
)
```

Rule 6744

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\int \left(2ab + 2b^2x^2\right)\sqrt{c + ex + dx^2}\, dx}{2ab + 2b^2x^2}$$

$$= \frac{bx\left(c + ex + dx^2\right)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{4d\left(a + bx^2\right)} + \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\int \left(-2b(bc - 4a\ldots\right)}{4d\left(2ab + 2\ldots\right)}$$

$$= -\frac{5be\left(c + ex + dx^2\right)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{24d^2\left(a + bx^2\right)} + \frac{bx\left(c + ex + dx^2\right)^{3/2}\sqrt{a^2 + 2abx^2}}{4d\left(a + bx^2\right)}$$

$$= -\frac{\left(4bcd - 16ad^2 - 5be^2\right)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{64d^3\left(a + bx^2\right)} - \frac{5be\left(c + \ldots\right)}{}$$

$$= -\frac{\left(4bcd - 16ad^2 - 5be^2\right)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{64d^3\left(a + bx^2\right)} - \frac{5be\left(c + \ldots\right)}{}$$

$$= -\frac{\left(4bcd - 16ad^2 - 5be^2\right)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{64d^3\left(a + bx^2\right)} - \frac{5be\left(c + \ldots\right)}{}$$

Mathematica [A]   time = 0.231036, size = 168, normalized size = 0.59

$$\frac{\sqrt{\left(a + bx^2\right)^2}\left(2\sqrt{d}\sqrt{c + x(dx + e)}\left(48ad^2(2dx + e) + b\left(4cd(6dx - 13e) + 8d^2ex^2 + 48d^3x^3 - 10de^2x + 15e^3\right)\right) - 3\left(4cd - e^2\ldots\right)}{384d^{7/2}\left(a + bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(48*a*d^2*(e + 2*d*x)
 + b*(15*e^3 - 10*d*e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3 + 4*c*d*(-13*e + 6*d*x
))) - 3*(4*c*d - e^2)*(4*b*c*d - 16*a*d^2 - 5*b*e^2)*ArcTanh[(e + 2*d*x)/(2
```

```
*Sqrt[d]*Sqrt[c + x*(e + d*x)])])]))/(384*d^(7/2)*(a + b*x^2))
```

Maple [A]   time = 0.008, size = 373, normalized size = 1.3

$$\frac{1}{384\,bx^2 + 384\,a}\sqrt{\left(bx^2 + a\right)^2}\left(96\,d^{7/2}\left(x^2d + ex + c\right)^{3/2}xb - 80\,d^{5/2}\left(x^2d + ex + c\right)^{3/2}be + 192\,d^{9/2}\sqrt{x^2d + ex + c}xa - 48\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/384*((b*x^2+a)^2)^(1/2)*(96*d^(7/2)*(d*x^2+e*x+c)^(3/2)*x*b-80*d^(5/2)*(d
*x^2+e*x+c)^(3/2)*b*e+192*d^(9/2)*(d*x^2+e*x+c)^(1/2)*x*a-48*d^(7/2)*(d*x^2
+e*x+c)^(1/2)*x*b*c+60*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b*e^2+96*d^(7/2)*(d*x^
2+e*x+c)^(1/2)*a*e-24*d^(5/2)*(d*x^2+e*x+c)^(1/2)*b*c*e+30*d^(3/2)*(d*x^2+e
*x+c)^(1/2)*b*e^3+192*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2
))*a*c*d^4-48*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*a*d^3
*e^2-48*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*c^2*d^3+7
2*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*c*d^2*e^2-15*ln
(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e^4)/(b*x^2+a)/d^
(9/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + ex + c}\sqrt{\left(bx^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x^2 + a)^2), x)
```

Fricas [A]   time = 1.92417, size = 857, normalized size = 3.03

$$\left[\frac{3\left(16\,bc^2d^2 - 64\,acd^3 + 5\,be^4 - 8\left(3\,bcd - 2\,ad^2\right)e^2\right)\sqrt{d}\log\left(8\,d^2x^2 + 8\,dex - 4\sqrt{dx^2 + ex + c}(2\,dx + e)\sqrt{d} + 4\,cd + e^2\right)}{768\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*b*c^2*d^2 - 64*a*c*d^3 + 5*b*e^4 - 8*(3*b*c*d - 2*a*d^2)*e^2)
*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt
(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 8*b*d^3*e*x^2 + 15*b*d*e^3 - 4*(13*b
*c*d^2 - 12*a*d^3)*e + 2*(12*b*c*d^3 + 48*a*d^4 - 5*b*d^2*e^2)*x)*sqrt(d*x^
2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 - 64*a*c*d^3 + 5*b*e^4 - 8*(3*b*c
*d - 2*a*d^2)*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sq
rt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x^3 + 8*b*d^3*e*x^2 + 15*b*d*
```

```
e^3 - 4*(13*b*c*d^2 - 12*a*d^3)*e + 2*(12*b*c*d^3 + 48*a*d^4 - 5*b*d^2*e^2)
*x)*sqrt(d*x^2 + e*x + c))/d^4]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]   time = 1.15152, size = 358, normalized size = 1.27

$$\frac{1}{192} \sqrt{dx^2 + xe + c} \left(2 \left(4 \left(6\,bx\mathrm{sgn}\left(bx^2 + a\right) + \frac{be\mathrm{sgn}\left(bx^2 + a\right)}{d} \right) x + \frac{12\,bcd^2\mathrm{sgn}\left(bx^2 + a\right) + 48\,ad^3\mathrm{sgn}\left(bx^2 + a\right) - 5\,bde^2\mathrm{sg}}{d^3} \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(d*x^2 + x*e + c)*(2*(4*(6*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^2 + a
)/d)*x + (12*b*c*d^2*sgn(b*x^2 + a) + 48*a*d^3*sgn(b*x^2 + a) - 5*b*d*e^2*s
gn(b*x^2 + a))/d^3)*x - (52*b*c*d*e*sgn(b*x^2 + a) - 48*a*d^2*e*sgn(b*x^2 +
 a) - 15*b*e^3*sgn(b*x^2 + a))/d^3) + 1/128*(16*b*c^2*d^2*sgn(b*x^2 + a) -
64*a*c*d^3*sgn(b*x^2 + a) - 24*b*c*d*e^2*sgn(b*x^2 + a) + 16*a*d^2*e^2*sgn(
b*x^2 + a) + 5*b*e^4*sgn(b*x^2 + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x
*e + c))*sqrt(d) - e))/d^(7/2)
```

3.39 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-be^2\right)}{8d^2\left(a+bx^2\right)}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}\left(8ad^2-b\left(4cd-e^2\right)\right)\tanh^{-1}\left(\frac{2dx-}{2\sqrt{d}\sqrt{c+}}\right)}{16d^{5/2}\left(a+bx^2\right)}$$

```
[Out] ((8*a*d^2 - b*e^2 - 2*b*d*e*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(8*d^2*(a + b*x^2)) + (b*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (e*(8*a*d^2 - b*(4*c*d - e^2))*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x
^2])])/(16*d^(5/2)*(a + b*x^2)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x^2)
```

Rubi [A]   time = 0.75443, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6744, 1653, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}\left(8ad^2-2bdex-be^2\right)}{8d^2\left(a+bx^2\right)}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}\left(8ad^2-b\left(4cd-e^2\right)\right)\tanh^{-1}\left(\frac{2dx-}{2\sqrt{d}\sqrt{c+}}\right)}{16d^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] ((8*a*d^2 - b*e^2 - 2*b*d*e*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(8*d^2*(a + b*x^2)) + (b*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (e*(8*a*d^2 - b*(4*c*d - e^2))*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x
^2])])/(16*d^(5/2)*(a + b*x^2)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x^2)
```

Rule 6744

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(2ab+2b^2x^2)\sqrt{c+ex+dx^2}}{x}\,dx}{2ab+2b^2x^2}$$

$$= \frac{b\left(c+ex+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(6abd-3b^2ex}{3d\left(2ab+2b^2x^2\right)}$$

$$= \frac{\left(8ad^2-be^2-2bdex\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2\left(a+bx^2\right)} + \frac{b\left(c+ex+dx^2\right.}{3d}$$

$$= \frac{\left(8ad^2-be^2-2bdex\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2\left(a+bx^2\right)} + \frac{b\left(c+ex+dx^2\right.}{3d}$$

$$= \frac{\left(8ad^2-be^2-2bdex\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2\left(a+bx^2\right)} + \frac{b\left(c+ex+dx^2\right.}{3d}$$

$$= \frac{\left(8ad^2-be^2-2bdex\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2\left(a+bx^2\right)} + \frac{b\left(c+ex+dx^2\right.}{3d}$$

Mathematica [A]   time = 0.322493, size = 176, normalized size = 0.62

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(2\sqrt{d}\left(\sqrt{c+x(dx+e)}\left(24ad^2+b\left(8cd+8d^2x^2+2dex-3e^2\right)\right)-24a\sqrt{c}d^2\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right)\right)+3e\left(8\right.}{48d^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]


[Out] (Sqrt[(a + b*x^2)^2]*(3*e*(8*a*d^2 + b*(-4*c*d + e^2))*ArcTanh[(e + 2*d*x)/
(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(24*a
*d^2 + b*(8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) - 24*a*Sqrt[c]*d^2*ArcTanh[
(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]))))/(48*d^(5/2)*(a + b*x^2))
```

Maple [A]   time = 0.007, size = 251, normalized size = 0.9

$$-\frac{1}{48\,bx^2+48\,a}\sqrt{\left(bx^2+a\right)^2}\left(48\,d^{7/2}\ln\left(\frac{2\,c+ex+2\,\sqrt{c}\sqrt{x^2d+ex+c}}{x}\right)\sqrt{c}\,a-16\,d^{5/2}\left(x^2d+ex+c\right)^{3/2}b+12\,d^{5/2}\sqrt{x^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x)


[Out] -1/48*((b*x^2+a)^2)^(1/2)*(48*d^(7/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(
1/2))/x)*c^(1/2)*a-16*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b+12*d^(5/2)*(d*x^2+e*x+c
)^(1/2)*x*b*e-48*d^(7/2)*(d*x^2+e*x+c)^(1/2)*a+6*d^(3/2)*(d*x^2+e*x+c)^(1/2
)*b*e^2-24*d^3*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*a*e+
12*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*c*d^2*e-3*ln(1
/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e^3)/(b*x^2+a)/d^(7
/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + ex + c}\sqrt{\left(bx^2 + a\right)^2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x^2 + a)^2)/x, x)
```

Fricas [A]   time = 7.15379, size = 1798, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a*sqrt(c)*d^3*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e
*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*
e)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sq
rt(d) + 4*c*d + e^2) + 4*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3
- 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3, 1/48*(24*a*sqrt(c)*d^3*log((8*c*e*
x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2
)/x^2) - 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*e)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 +
 e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(8*b*d^3*x^2 +
2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3 - 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3,
 1/96*(96*a*sqrt(-c)*d^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
-c)/(c*d*x^2 + c*e*x + c^2)) + 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*e)*sqrt(d)*lo
g(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d
 + e^2) + 4*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3 - 3*b*d*e^2)*
sqrt(d*x^2 + e*x + c))/d^3, 1/48*(48*a*sqrt(-c)*d^3*arctan(1/2*sqrt(d*x^2 +
 e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 3*(b*e^3 - 4*(b*c
*d - 2*a*d^2)*e)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt
(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 +
24*a*d^3 - 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.40 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=294

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(8ad^2+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}\left(a+bx^2\right)} - \frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2+ex\right)^{3/2}}{cx\left(a+bx^2\right)} + \frac{\sqrt{a^2+2abx^2}}{}$$

```
[Out] (((b*c + 4*a*d)*e + 2*d*(b*c + 2*a*d)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*c*d*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a + b*x^2)) + ((4*b*c*d + 8*a*d^2 - b*e^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e
*x + d*x^2])])/(8*d^(3/2)*(a + b*x^2)) - (a*e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[c]*(a +
b*x^2))
```

Rubi [A]   time = 0.767118, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6744, 1650, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(8ad^2+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}\left(a+bx^2\right)} - \frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2+ex\right)^{3/2}}{cx\left(a+bx^2\right)} + \frac{\sqrt{a^2+2abx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (((b*c + 4*a*d)*e + 2*d*(b*c + 2*a*d)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*c*d*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a + b*x^2)) + ((4*b*c*d + 8*a*d^2 - b*e^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e
*x + d*x^2])])/(8*d^(3/2)*(a + b*x^2)) - (a*e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[c]*(a +
b*x^2))
```

Rule 6744

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^2}\, dx = \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\int \frac{\left(2ab + 2b^2x^2\right)\sqrt{c + ex + dx^2}}{x^2}\, dx}{2ab + 2b^2x^2}$$

$$= -\frac{a\left(c + ex + dx^2\right)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{cx\left(a + bx^2\right)} - \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\int \frac{(-abe - 2b(bc + 2}{c\left(2ab + 2b^2x^2\right)}}$$

$$= \frac{((bc + 4ad)e + 2d(bc + 2ad)x)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{4cd\left(a + bx^2\right)} - \frac{a\left(c + ex +}{4cd\left(a + bx^2\right)}$$

$$= \frac{((bc + 4ad)e + 2d(bc + 2ad)x)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{4cd\left(a + bx^2\right)} - \frac{a\left(c + ex +}{4cd\left(a + bx^2\right)}$$

$$= \frac{((bc + 4ad)e + 2d(bc + 2ad)x)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{4cd\left(a + bx^2\right)} - \frac{a\left(c + ex +}{4cd\left(a + bx^2\right)}$$

$$= \frac{((bc + 4ad)e + 2d(bc + 2ad)x)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{4cd\left(a + bx^2\right)} - \frac{a\left(c + ex +}{4cd\left(a + bx^2\right)}$$

Mathematica [A]   time = 0.261252, size = 171, normalized size = 0.58

$$\frac{\sqrt{\left(a + bx^2\right)^2}\left(\sqrt{c}x\left(8ad^2 + 4bcd - be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right) + 2\sqrt{d}\left(\sqrt{c}\sqrt{c + x(dx + e)}(bx(2dx + e) - 4ad) - 2adex\tanh^{-1}\right.}{8\sqrt{c}d^{3/2}x\left(a + bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(4*b*c*d + 8*a*d^2 - b*e^2)*x*ArcTanh[(e + 2*
d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c]*Sqrt[c + x*(e
+ d*x)]*(-4*a*d + b*x*(e + 2*d*x)) - 2*a*d*e*x*ArcTanh[(2*c + e*x)/(2*Sqrt[
c]*Sqrt[c + x*(e + d*x)])]))))/(8*Sqrt[c]*d^(3/2)*x*(a + b*x^2))
```

Maple [A]   time = 0.011, size = 289, normalized size = 1.

$$\frac{1}{\left(8\,bx^2 + 8\,a\right)cx}\sqrt{\left(bx^2 + a\right)^2}\left(8\,d^{7/2}\sqrt{x^2d + ex + c}\,x^2a - 4\,d^{5/2}\sqrt{c}\ln\left(\frac{2\,c + ex + 2\,\sqrt{c}\sqrt{x^2d + ex + c}}{x}\right)xae + 4\,d^{5/2}\sqrt{x^2d + ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] 1/8*((b*x^2+a)^2)^(1/2)*(8*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x^2*a-4*d^(5/2)*c^(1
/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*x*a*e+4*d^(5/2)*(d*x^2+e*
x+c)^(1/2)*x^2*b*c-8*d^(5/2)*(d*x^2+e*x+c)^(3/2)*a+8*d^(5/2)*(d*x^2+e*x+c)^
(1/2)*x*a*e+2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x*b*c*e+8*ln(1/2*(2*(d*x^2+e*x+c)
^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*x*a*c*d^3+4*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*
d^(1/2)+2*d*x+e)/d^(1/2))*x*b*c^2*d^2-ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)
+2*d*x+e)/d^(1/2))*x*b*c*d*e^2)/(b*x^2+a)/c/x/d^(5/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + ex + c}\sqrt{\left(bx^2 + a\right)^2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x^2 + a)^2)/x^2, x)
```

Fricas [A]   time = 6.28716, size = 1731, normalized size = 5.89

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] [1/16*(4*a*sqrt(c)*d^2*e*x*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2
+ e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c^2*d + 8*a*c*d^2 - b*c
*e^2)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(d) + 4*c*d + e^2) + 4*(2*b*c*d^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(
d*x^2 + e*x + c))/(c*d^2*x), 1/8*(2*a*sqrt(c)*d^2*e*x*log((8*c*e*x + (4*c*d
 + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (
4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c
)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*c*d^2*x^2 + b*c*d*
e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*x), 1/16*(8*a*sqrt(-c)*d^2*e
*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x +
 c^2)) - (4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*
x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(2*b*c*d
^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*x), 1/8*(4*a*
sqrt(-c)*d^2*e*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d
*x^2 + c*e*x + c^2)) - (4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(-d)*x*arctan(
1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2
*(2*b*c*d^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*x)]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

<div align="center">Timed out</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.41 $\int \dfrac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=288

$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(4acd-ae^2+8bc^2\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}\left(a+bx^2\right)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2+ex\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2\cdots}}{\cdots}$$

```
[Out] ((a*e + 2*(2*b*c + a*d)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*c*x*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(2*c*x^2*(a + b*x^2)) + (b*e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x
^2)) - ((8*b*c^2 + 4*a*c*d - a*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh
[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x^2))
```

Rubi [A]   time = 0.784178, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6744, 1650, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(4acd-ae^2+8bc^2\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}\left(a+bx^2\right)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2+ex\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((a*e + 2*(2*b*c + a*d)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*c*x*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(2*c*x^2*(a + b*x^2)) + (b*e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x
^2)) - ((8*b*c^2 + 4*a*c*d - a*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh
[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x^2))
```

Rule 6744

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
```

```
 - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(2ab+2b^2x^2\right)\sqrt{c+ex+dx^2}}{x^3}\,dx}{2ab+2b^2x^2}$$

$$= -\frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(abe-2b(2bc+a}{2c\left(2ab+2b^2x^2\right)}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{}}{2cx^2\left(a-}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{}}{2cx^2\left(a-}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{}}{2cx^2\left(a-}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{}}{2cx^2\left(a-}$$

Mathematica [A]   time = 0.358819, size = 177, normalized size = 0.61

$$\frac{\sqrt{\left(a + bx^2\right)^2}\left(4bc^{3/2}ex^2\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right) - \sqrt{d}\left(x^2\left(4acd - ae^2 + 8bc^2\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right) + 2\sqrt{c}\sqrt{c + x(dx}\right.}{8c^{3/2}\sqrt{d}x^2\left(a + bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(4*b*c^(3/2)*e*x^2*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt
[c + x*(e + d*x)])] - Sqrt[d]*(2*Sqrt[c]*(2*a*c + a*e*x - 4*b*c*x^2)*Sqrt[c
 + x*(e + d*x)] + (8*b*c^2 + 4*a*c*d - a*e^2)*x^2*ArcTanh[(2*c + e*x)/(2*Sq
rt[c]*Sqrt[c + x*(e + d*x)])])))/(8*c^(3/2)*Sqrt[d]*x^2*(a + b*x^2))
```

Maple [A]   time = 0.01, size = 329, normalized size = 1.1

$$\frac{1}{8\,x^2c^2\left(bx^2 + a\right)}\sqrt{\left(bx^2 + a\right)^2}\left(-4\,d^{5/2}c^{3/2}\ln\left(\frac{2\,c + ex + 2\,\sqrt{c}\sqrt{x^2d + ex + c}}{x}\right)x^2a - 8\,d^{3/2}c^{5/2}\ln\left(\frac{2\,c + ex + 2\,\sqrt{c}\sqrt{x^2d +}}{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] 1/8*((b*x^2+a)^2)^(1/2)*(-4*d^(5/2)*c^(3/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*
x+c)^(1/2))/x)*x^2*a-8*d^(3/2)*c^(5/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^
(1/2))/x)*x^2*b-2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^3*a*e+4*d^(5/2)*(d*x^2+e*x+
c)^(1/2)*x^2*a*c+d^(3/2)*c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))
/x)*x^2*a*e^2+2*d^(3/2)*(d*x^2+e*x+c)^(3/2)*x*a*e-2*d^(3/2)*(d*x^2+e*x+c)^(
1/2)*x^2*a*e^2+8*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^2*b*c^2+4*ln(1/2*(2*(d*x^2+e
*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d*x^2*b*c^2*e-4*d^(3/2)*(d*x^2+e*x+c)
^(3/2)*a*c)/d^(3/2)/x^2/c^2/(b*x^2+a)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + ex + c}\sqrt{\left(bx^2 + a\right)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x^2 + a)^2)/x^3, x)
```

Fricas [A]   time = 5.94203, size = 1766, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/16*(4*b*c^2*sqrt(d)*e*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (8*b*c^2*d + 4*a*c*d^2 - a*d*e^2)*
sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x
 + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*s
qrt(d*x^2 + e*x + c))/(c^2*d*x^2), -1/16*(8*b*c^2*sqrt(-d)*e*x^2*arctan(1/2
*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (8*b
*c^2*d + 4*a*c*d^2 - a*d*e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2
+ 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(4*b*c^2*d*
x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2 + e*x + c))/(c^2*d*x^2), 1/8*(2*b*c
^2*sqrt(d)*e*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x +
 e)*sqrt(d) + 4*c*d + e^2) + (8*b*c^2*d + 4*a*c*d^2 - a*d*e^2)*sqrt(-c)*x^2
*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c
^2)) + 2*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2 + e*x + c))/(c^
2*d*x^2), -1/8*(4*b*c^2*sqrt(-d)*e*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*
d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (8*b*c^2*d + 4*a*c*d^2 - a*d*e
^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d
*x^2 + c*e*x + c^2)) - 2*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2
 + e*x + c))/(c^2*d*x^2)]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [A]   time = 1.23649, size = 500, normalized size = 1.74

$$-\frac{be \log\left(\left|-2\left(\sqrt{d}x - \sqrt{dx^2 + xe + c}\right)d - \sqrt{d}e\right|\right)\operatorname{sgn}\left(bx^2 + a\right)}{2\sqrt{d}} + \sqrt{dx^2 + xe + c}\,b\operatorname{sgn}\left(bx^2 + a\right) + \frac{\left(8\,bc^2\operatorname{sgn}\left(bx^2 + a\right) + 4\,ac}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*e*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*d - sqrt(d)*e))*sgn
(b*x^2 + a)/sqrt(d) + sqrt(d*x^2 + x*e + c)*b*sgn(b*x^2 + a) + 1/4*(8*b*c^2
*sgn(b*x^2 + a) + 4*a*c*d*sgn(b*x^2 + a) - a*e^2*sgn(b*x^2 + a))*arctan(-(s
qrt(d)*x - sqrt(d*x^2 + x*e + c))/sqrt(-c))/(sqrt(-c)*c) + 1/4*(4*(sqrt(d)*
x - sqrt(d*x^2 + x*e + c))^3*a*c*d*sgn(b*x^2 + a) + 8*(sqrt(d)*x - sqrt(d*x
^2 + x*e + c))^2*a*c*sqrt(d)*e*sgn(b*x^2 + a) + 4*(sqrt(d)*x - sqrt(d*x^2 +
 x*e + c))*a*c^2*d*sgn(b*x^2 + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a
*e^2*sgn(b*x^2 + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))*a*c*e^2*sgn(b*x^2
 + a))/(((sqrt(d)*x - sqrt(d*x^2 + x*e + c))^2 - c)^2*c)
```

3.42 $\displaystyle\int \frac{\sqrt{c+ex+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$

Optimal. Leaf size=294

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2+ex}\left(2ace-x\left(8bc^2-ae^2\right)\right)}{8c^2x^2\left(a+bx^2\right)} - \frac{e\sqrt{a^2+2abx^2+b^2x^4}\left(8bc^2-a\left(4cd-e^2\right)\right)\tanh^{-1}\left(\frac{2c}{2\sqrt{c}\sqrt{c}}\right)}{16c^{5/2}\left(a+bx^2\right)}$$

```
[Out] ((2*a*c*e - (8*b*c^2 - a*e^2)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(8*c^2*x^2*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(3*c*x^3*(a + b*x^2)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a
+ b*x^2) - (e*(8*b*c^2 - a*(4*c*d - e^2))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*A
rcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(16*c^(5/2)*(a + b*x
^2))
```

Rubi [A]   time = 0.786889, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6744, 1650, 810, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2+ex}\left(2ace-x\left(8bc^2-ae^2\right)\right)}{8c^2x^2\left(a+bx^2\right)} - \frac{e\sqrt{a^2+2abx^2+b^2x^4}\left(8bc^2-a\left(4cd-e^2\right)\right)\tanh^{-1}\left(\frac{2c}{2\sqrt{c}\sqrt{c}}\right)}{16c^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^4,x]
```

```
[Out] ((2*a*c*e - (8*b*c^2 - a*e^2)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(8*c^2*x^2*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(3*c*x^3*(a + b*x^2)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a
+ b*x^2) - (e*(8*b*c^2 - a*(4*c*d - e^2))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*A
rcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(16*c^(5/2)*(a + b*x
^2))
```

Rule 6744

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2)))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(2ab+2b^2x^2)\sqrt{c+ex+dx^2}}{x^4}\,dx}{2ab+2b^2x^2}$$

$$= -\frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3\left(a+bx^2\right)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(3abe-6b^2}{3c\left(2ab+2b^2x^2\right)}$$

$$= \frac{\left(2ace-\left(8bc^2-ae^2\right)x\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2\left(a+bx^2\right)} - \frac{a\left(c+ex+dx\right)}{3}$$

$$= \frac{\left(2ace-\left(8bc^2-ae^2\right)x\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2\left(a+bx^2\right)} - \frac{a\left(c+ex+dx\right)}{3}$$

$$= \frac{\left(2ace-\left(8bc^2-ae^2\right)x\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2\left(a+bx^2\right)} - \frac{a\left(c+ex+dx\right)}{3}$$

$$= \frac{\left(2ace-\left(8bc^2-ae^2\right)x\right)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2\left(a+bx^2\right)} - \frac{a\left(c+ex+dx\right)}{3}$$

Mathematica [A]   time = 0.292935, size = 189, normalized size = 0.64

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(-2\sqrt{c}\sqrt{c+x(dx+e)}\left(a\left(8c^2+2cx(4dx+e)-3e^2x^2\right)+24bc^2x^2\right)-3ex^3\left(a\left(e^2-4cd\right)+8bc^2\right)\tanh^{-1}\left(\frac{}{2}\right)\right.}{48c^{5/2}x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*Sqrt[c]*Sqrt[c + x*(e + d*x)]*(24*b*c^2*x^2 + a*(8
*c^2 - 3*e^2*x^2 + 2*c*x*(e + 4*d*x))) + 48*b*c^(5/2)*Sqrt[d]*x^3*ArcTanh[(
e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] - 3*e*(8*b*c^2 + a*(-4*c*d +
e^2))*x^3*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])])))/(48*c^(5
/2)*x^3*(a + b*x^2))
```

Maple [A]   time = 0.011, size = 412, normalized size = 1.4

$$\frac{1}{48\,x^3c^3\left(bx^2+a\right)}\sqrt{\left(bx^2+a\right)^2}\left(12\,d^{5/2}c^{3/2}\ln\left(\frac{2c+ex+2\sqrt{c}\sqrt{x^2d+ex+c}}{x}\right)x^3ae-24\,d^{3/2}c^{5/2}\ln\left(\frac{2c+ex+2\sqrt{c}\sqrt{x^2}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x)
```

```
[Out] 1/48*((b*x^2+a)^2)^(1/2)*(12*d^(5/2)*c^(3/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e
*x+c)^(1/2))/x)*x^3*a*e-24*d^(3/2)*c^(5/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x
+c)^(1/2))/x)*x^3*b*e+6*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^4*a*e^2+48*d^(5/2)*(d
*x^2+e*x+c)^(1/2)*x^4*b*c^2-12*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^3*a*c*e-3*d^(3
/2)*c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*x^3*a*e^3-6*d^(3/
2)*(d*x^2+e*x+c)^(3/2)*x^2*a*e^2-48*d^(3/2)*(d*x^2+e*x+c)^(3/2)*x^2*b*c^2+6
*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^3*a*e^3+48*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^3*b
```

```
*c^2*e+48*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*x^3*b*c^3
*d^2+12*d^(3/2)*(d*x^2+e*x+c)^(3/2)*x*a*c*e-16*d^(3/2)*(d*x^2+e*x+c)^(3/2)*
a*c^2)/d^(3/2)/x^3/c^3/(b*x^2+a)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+ex+c}\sqrt{\left(bx^2+a\right)^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x^2 + a)^2)/x^4, x)
```

Fricas [A]   time = 6.50311, size = 1887, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/96*(48*b*c^3*sqrt(d)*x^3*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x +
c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*s
qrt(c)*x^3*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x
+ 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c
^2*d - 3*a*c*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3), -1/96*(96*b*c^3*sq
rt(-d)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 +
 d*e*x + c*d)) - 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt(c)*x^3*log((8*c*e*x
 + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)
/x^2) + 4*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e^2)*x^2)*
sqrt(d*x^2 + e*x + c))/(c^3*x^3), 1/48*(24*b*c^3*sqrt(d)*x^3*log(8*d^2*x^2
+ 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 3*
(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt(-c)*x^3*arctan(1/2*sqrt(d*x^2 + e*x +
c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2*e*x + 8*a*c^3
 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3)
, -1/48*(48*b*c^3*sqrt(-d)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)
*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt
(-c)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c
*e*x + c^2)) + 2*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e^2
)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3)]
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**4,x)
```

[Out] Timed out

Giac [B]   time = 1.47226, size = 972, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) + e))*sgn(
b*x^2 + a) + 1/8*(8*b*c^2*e*sgn(b*x^2 + a) - 4*a*c*d*e*sgn(b*x^2 + a) + a*e
^3*sgn(b*x^2 + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + x*e + c))/sqrt(-c))/(s
qrt(-c)*c^2) + 1/24*(24*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^5*b*c^2*sqrt(d)
*e*sgn(b*x^2 + a) + 12*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^5*a*c*d^(3/2)*e*
sgn(b*x^2 + a) + 48*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^4*b*c^3*d*sgn(b*x^2
 + a) + 48*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^4*a*c^2*d^2*sgn(b*x^2 + a) -
 48*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*b*c^3*sqrt(d)*e*sgn(b*x^2 + a) +
48*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*c^2*d^(3/2)*e*sgn(b*x^2 + a) - 9
6*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^2*b*c^4*d*sgn(b*x^2 + a) - 3*(sqrt(d)
*x - sqrt(d*x^2 + x*e + c))^5*a*sqrt(d)*e^3*sgn(b*x^2 + a) + 24*(sqrt(d)*x
- sqrt(d*x^2 + x*e + c))*b*c^4*sqrt(d)*e*sgn(b*x^2 + a) + 36*(sqrt(d)*x - s
qrt(d*x^2 + x*e + c))*a*c^3*d^(3/2)*e*sgn(b*x^2 + a) + 48*b*c^5*d*sgn(b*x^2
 + a) + 16*a*c^4*d^2*sgn(b*x^2 + a) + 48*(sqrt(d)*x - sqrt(d*x^2 + x*e + c)
)^2*a*c^2*d*e^2*sgn(b*x^2 + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*
c*sqrt(d)*e^3*sgn(b*x^2 + a) + 3*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*a*c^2*
sqrt(d)*e^3*sgn(b*x^2 + a))/(((sqrt(d)*x - sqrt(d*x^2 + x*e + c))^2 - c)^3*
c^2*sqrt(d))
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1  (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2  (* ::Package:: *)
3
4  (* ::Subsection:: *)
5  (*GradeAntiderivative[result,optimal]*)
6
7
8  (* ::Text:: *)
9  (*If result and optimal are mathematical expressions, *)
10 (*          GradeAntiderivative[result,optimal] returns*)
11 (*  "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (*  "C" if result involves higher level functions than necessary*)
14 (*  "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (*  "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24       "B"],
25     "C"],
26   If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28   "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

```

```
101   AppellFunctionQ[func_] :=
102      MemberQ[{AppellF1},func]
```

4.0.2   Maple grading function

```
1   # File: GradeAntiderivative.mpl
2   # Original version thanks to Albert Rich emailed on 03/21/2017
3
4   #Nasser 03/22/2017  Use Maple leaf count instead since buildin
5   #Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
6   #Nasser 03/24/2017  corrected the check for complex result
7   #Nasser 10/27/2017  check for leafsize and do not call ExpnType()
8   #                   if leaf size is "too large". Set at 500,000
9   #Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
10  #                   see problem 156, file Apostol_Problems
11
12  GradeAntiderivative := proc(result,optimal)
13  local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
        debug:=false;
14
15      leaf_count_result:=leafcount(result);
16      #do NOT call ExpnType() if leaf size is too large. Recursion problem
17      if leaf_count_result > 500000 then
18          return "B";
19      fi;
20
21      leaf_count_optimal:=leafcount(optimal);
22
23      ExpnType_result:=ExpnType(result);
24      ExpnType_optimal:=ExpnType(optimal);
25
26      if debug then
27          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
        ExpnType_optimal);
28      fi;
29
30  # If result and optimal are mathematical expressions,
31  #  GradeAntiderivative[result,optimal] returns
32  #   "F" if the result fails to integrate an expression that
33  #       is integrable
34  #   "C" if result involves higher level functions than necessary
35  #   "B" if result is more than twice the size of the optimal
36  #       antiderivative
37  #   "A" if result can be considered optimal
38
39    #This check below actually is not needed, since I only
40    #call this grading only for passed integrals. i.e. I check
41    #for "F" before calling this. But no harm of keeping it here.
42    #just in case.
43
44
45    if not type(result,freeof('int')) then
46        return "F";
47    end if;
48
49
50    if ExpnType_result<=ExpnType_optimal then
51        if debug then
52            print("ExpnType_result<=ExpnType_optimal");
53        fi;
54        if is_contains_complex(result) then
55            if is_contains_complex(optimal) then
56                if debug then
```

```
57              print("both result and optimal complex");
58          fi;
59          #both result and optimal complex
60          if leaf_count_result<=2*leaf_count_optimal then
61              return "A";
62          else
63              return "B";
64          end if
65      else #result contains complex but optimal is not
66          if debug then
67              print("result contains complex but optimal is not");
68          fi;
69          return "C";
70      end if
71   else # result do not contain complex
72          # this assumes optimal do not as well
73          if debug then
74              print("result do not contain complex, this assumes optimal do
    not as well");
75          fi;
76          if leaf_count_result<=2*leaf_count_optimal then
77              if debug then
78                  print("leaf_count_result<=2*leaf_count_optimal");
79              fi;
80              return "A";
81          else
82              if debug then
83                  print("leaf_count_result>2*leaf_count_optimal");
84              fi;
85              return "B";
86          end if
87      end if
88   else #ExpnType(result) > ExpnType(optimal)
89      if debug then
90          print("ExpnType(result) > ExpnType(optimal)");
91      fi;
92      return "C";
93   end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103    return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
```

```
119    if type(expn,'atomic') then
120       1
121    elif type(expn,'list') then
122       apply(max,map(ExpnType,expn))
123    elif type(expn,'sqrt') then
124       if type(op(1,expn),'rational') then
125          1
126       else
127          max(2,ExpnType(op(1,expn)))
128       end if
129    elif type(expn,'`^`') then
130       if type(op(2,expn),'integer') then
131          ExpnType(op(1,expn))
132       elif type(op(2,expn),'rational') then
133          if type(op(1,expn),'rational') then
134             1
135          else
136             max(2,ExpnType(op(1,expn)))
137          end if
138       else
139             max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140       end if
141    elif type(expn,'`+`') or type(expn,'`*`') then
142       max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143    elif ElementaryFunctionQ(op(0,expn)) then
144       max(3,ExpnType(op(1,expn)))
145    elif SpecialFunctionQ(op(0,expn)) then
146       max(4,apply(max,map(ExpnType,[op(expn)])))
147    elif HypergeometricFunctionQ(op(0,expn)) then
148       max(5,apply(max,map(ExpnType,[op(expn)])))
149    elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151    elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153    9
154    end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159    member(func,[
160          exp,log,ln,
161          sin,cos,tan,cot,sec,csc,
162          arcsin,arccos,arctan,arccot,arcsec,arccsc,
163          sinh,cosh,tanh,coth,sech,csch,
164          arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168    member(func,[
169          erf,erfc,erfi,
170          FresnelS,FresnelC,
171          Ei,Ei,Li,Si,Ci,Shi,Chi,
172          GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173          EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177    member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181    member(func,[AppellF1])
```

```
182   end proc:
183
184   # u is a sum or product.  rest(u) returns all but the
185   # first term or factor of u.
186   rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192   end proc:
193
194   #leafcount(u) returns the number of nodes in u.
195   #Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
196   leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198   end proc:
```

4.0.3   Sympy grading function

```
1    #Dec 24, 2019. Nasser M. Abbasi:
2    #               Port of original Maple grading function by
3    #               Albert Rich to use with Sympy/Python
4    #Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
5    #               added 'exp_polar'
6    from sympy import *
7
8    def leaf_count(expr):
9        #sympy do not have leaf count function. This is approximation
10       return round(1.7*count_ops(expr))
11
12   def is_sqrt(expr):
13       if isinstance(expr,Pow):
14           if expr.args[1] == Rational(1,2):
15               return True
16           else:
17               return False
18       else:
19           return False
20
21   def is_elementary_function(func):
22       return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23               asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24               asinh,acosh,atanh,acoth,asech,acsch
25           ]
26
27   def is_special_function(func):
28       return func in [ erf,erfc,erfi,
29               fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30               gamma,loggamma,digamma,zeta,polylog,LambertW,
31               elliptic_f,elliptic_e,elliptic_pi,exp_polar
32           ]
33
34   def is_hypergeometric_function(func):
35       return func in [hyper]
36
37   def is_appell_function(func):
38       return func in [appellf1]
39
40   def is_atom(expn):
41       try:
42           if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43               return True
```

```python
44              else:
45                  return False
46
47      except AttributeError as error:
48          return False
49
50  def expnType(expn):
51      debug=False
52      if debug:
53          print("expn=",expn,"type(expn)=",type(expn))
54
55      if is_atom(expn):
56          return 1
57      elif isinstance(expn,list):
58          return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
59      elif  is_sqrt(expn):
60          if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61              return 1
62          else:
63              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
64      elif isinstance(expn,Pow):    #type(expn,'`^`')
65          if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
66              return expnType(expn.args[0])    #ExpnType(op(1,expn))
67          elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
68              if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                  return 1
70              else:
71                  return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
      )))
72          else:
73              return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
      ExpnType(op(1,expn)),ExpnType(op(2,expn)))
74      elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
      type(expn,'`*`')
75          m1 = expnType(expn.args[0])
76          m2 = expnType(list(expn.args[1:]))
77          return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
78      elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
79          return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
80      elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
81          m1 = max(map(expnType, list(expn.args)))
82          return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
83      elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
      expn))
84          m1 = max(map(expnType, list(expn.args)))
85          return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86      elif is_appell_function(expn.func):
87          m1 = max(map(expnType, list(expn.args)))
88          return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89      elif isinstance(expn,RootSum):
90          m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
      ,Apply[List,expn]],7]],
91          return max(7,m1)
92      elif str(expn).find("Integral") != -1:
93          m1 = max(map(expnType, list(expn.args)))
94          return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95      else:
96          return 9
97
98  #main function
99  def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
```

```
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
    well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"
```

4.0.4   SageMath grading function

```
1  #Dec 24, 2019. Nasser: Ported original Maple grading function by
2  #               Albert Rich to use with Sagemath. This is used to
3  #               grade Fricas, Giac and Maxima results.
4  #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5  #               'arctan2','floor','abs','log_integral'
6
7  from sage.all import *
8  from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()]+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34                    #since this estimate of leaf count is bit lower than
```

```
35                             #what it should be compared to Mathematica's
36
37  def is_sqrt(expr):
38      debug=False;
39      if expr.operator() == operator.pow:   #isinstance(expr,Pow):
40          if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41              if debug: print ("expr is sqrt")
42              return True
43          else:
44              return False
45      else:
46          return False
47
48  def is_elementary_function(func):
49      debug = False
50
51      m = func.name() in ['exp','log','ln',
52              'sin','cos','tan','cot','sec','csc',
53              'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54              'sinh','cosh','tanh','coth','sech','csch',
55              'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56          'arctan2','floor','abs'
57          ]
58      if debug:
59          if m:
60              print ("func ", func , " is elementary_function")
61          else:
62              print ("func ", func , " is NOT elementary_function")
63
64
65      return m
66
67  def is_special_function(func):
68      debug = False
69
70      if debug: print ("type(func)=", type(func))
71
72      m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73              'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74      sinh_integral'
74              'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
75              'polylog','lambert_w','elliptic_f','elliptic_e',
76              'elliptic_pi','exp_integral_e','log_integral']
77
78      if debug:
79          print ("m=",m)
80          if m:
81              print ("func ", func ," is special_function")
82          else:
83              print ("func ", func ," is NOT special_function")
84
85
86      return m
87
88
89  def is_hypergeometric_function(func):
90      return func.name() in ['hypergeometric','hypergeometric_M','
91      hypergeometric_U']
91
92  def is_appell_function(func):
93      return func.name() in ['hypergeometric']   #[appellf1] can't find this in
94      sagemath
94
```

```
95   def is_atom(expn):
96
97       #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
         sagemath-equivalent-to-atomic-type-in-maple/
98       try:
99           if expn.parent() is SR:
100              return expn.operator() is None
101          if expn.parent() in (ZZ, QQ, AA, QQbar):
102              return expn in expn.parent() # Should always return True
103          if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
         :
104              return expn in expn.parent().base_ring() or expn in expn.parent().
         gens()
105          return False
106
107      except AttributeError as error:
108          return False
109
110
111  def expnType(expn):
112      debug=False
113
114      if debug:
115          print (">>>>>Enter expnType, expn=", expn)
116          print (">>>>>is_atom(expn)=", is_atom(expn))
117
118      if is_atom(expn):
119          return 1
120      elif type(expn)==list:    #isinstance(expn,list):
121          return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122      elif  is_sqrt(expn):
123          if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
         Rational):
124              return 1
125          else:
126              return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
         args[0]))
127      elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128          if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
         )
129              return expnType(expn.operands()[0])   #expnType(expn.args[0])
130          elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
         Rational)
131              if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
         Rational)
132                  return 1
133              else:
134                  return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
         expn.args[0]))
135          else:
136              return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
         [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137      elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
         isinstance(expn,Add) or isinstance(expn,Mul)
138          m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139          m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140          return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141      elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
         .func)
142          return max(3,expnType(expn.operands()[0]))
143      elif is_special_function(expn.operator()): #is_special_function(expn.func)
144          m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
         expn.args)))
```

```
145            return max(4,m1)    #max(4,m1)
146        elif is_hypergeometric_function(expn.operator()): #
       is_hypergeometric_function(expn.func)
147            m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(
       expn.args)))
148            return max(5,m1)    #max(5,m1)
149        elif is_appell_function(expn.operator()):
150            m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(
       expn.args)))
151            return max(6,m1)    #max(6,m1)
152        elif str(expn).find("Integral") != -1: #this will never happen, since it
153                    #is checked before calling the grading function that is passed.
154                    #but kept it here.
155            m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(
       expn.args)))
156            return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157        else:
158            return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result  = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
       leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result  = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
       expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
       well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"
```